ANSWERS AND COMMENTS

QUESTION 1.1

1 W = 1 J s^{-1}, so the value of $L_\odot$ in terms of watts is simply $L_\odot = 3.84 \times 10^{26}$ W. Therefore the number of typical power stations required to match the Sun's energy output is given by

$$\frac{3.84 \times 10^{26}\ \text{W}}{2.5 \times 10^{9}\ \text{W}} = 1.54 \times 10^{17} \quad \text{to 3 significant figures}$$

So, to 2 significant figures, which correponds to the data in the question, the value is 1.5×10^{17} power stations.

In part, this question is intended to emphasize the immensity of the Sun's energy output, but an equally important purpose is to draw your attention to the use of SI units (the joule (J), the watt (W), etc.) and to the manipulation of powers of ten ($10^{26}/10^{9} = 10^{17}$, etc.) and the importance of working to the relevant number of significant figures.

QUESTION 1.2

It was stated earlier that the diameter of the photosphere is about 1.4×10^{6} km. The size of the sunspot can be estimated by simply measuring the diameter of the spot's image in Figure 1.1 and comparing this with the size of the image of the photosphere:

$$\text{sunspot diameter} = \frac{d_{\text{si}}}{d_{\text{pi}}} \times d_{\text{p}}$$

where d_{si} is the diameter of the sunspot's image, d_{pi} is the diameter of the photosphere's image, and d_{p} is the diameter of the photosphere. However, in using this formula it is important to remember that the Sun is a spherical body, so the further a sunspot is from the centre of the solar disc, the more it will be 'turned away' from the observer and the more its image will be foreshortened. Although it would be possible to make allowance for this foreshortening, it is much easier just to examine sunspot images near the centre of the disc, where the effect can be ignored. Such sunspot images seem to have a diameter that is about 2% of the diameter of the disc image. It follows that, for the large sunspots near the centre of Figure 1.1,

$$\text{sunspot diameter} \approx \frac{2}{100} \times 1.4 \times 10^{6}\ \text{km} = 2.8 \times 10^{4}\ \text{km}$$

So the estimated sunspot diameter is about 3×10^{4} km. It follows that large sunspots have diameters that are comparable to the diameter of the Earth (1.3×10^{4} km).

QUESTION 1.3

The small-scale structure in Figure 1.3b is partly due to the solar granulation seen in the photosphere. (Sunspots may also contribute.) The granules are bright whereas the lanes that separate them are dark. This variation shows up in the graph.

QUESTION 1.4

The frequencies are given by $f = c/\lambda$ (Equation 1.2, rearranged). So, for $\lambda = 400\,\text{nm}$,

$$f = \frac{3.00 \times 10^8\ \text{m s}^{-1}}{400 \times 10^{-9}\ \text{m}} = 7.50 \times 10^{14}\ \text{Hz}$$

and, for $\lambda = 700\,\text{nm}$,

$$f = \frac{3.00 \times 10^8\ \text{m s}^{-1}}{700 \times 10^{-9}\ \text{m}} = 4.29 \times 10^{14}\ \text{Hz}$$

Thus, the frequency range is from 4.29×10^{14} Hz to 7.50×10^{14} Hz.

QUESTION 1.5

The photon energies are given by $\varepsilon = hf$ (Equation 1.3). So, for $\lambda = 400\,\text{nm}$,

$$\varepsilon = (6.63 \times 10^{-34}\ \text{J s}) \times (7.50 \times 10^{14}\ \text{s}^{-1}) = 4.97 \times 10^{-19}\ \text{J}$$

Since 1 eV = 1.602×10^{-19} J, the photon energy in electronvolts is

$$\varepsilon = (4.97 \times 10^{-19}\ \text{J})/(1.602 \times 10^{-19}\ \text{J eV}^{-1}) = 3.10\ \text{eV}$$

(Alternatively, the energy in electronvolts can be obtained from Equation 1.3 and using a value of the Planck constant of $h = 4.14 \times 10^{-15}$ eV s.)

For $\lambda = 700\,\text{nm}$,

$$\varepsilon = (6.63 \times 10^{-34}\ \text{J s}) \times (4.29 \times 10^{14}\ \text{s}^{-1}) = 2.84 \times 10^{-19}\ \text{J}$$

and in electronvolts

$$\varepsilon = (2.84 \times 10^{-19}\ \text{J})/(1.602 \times 10^{-19}\ \text{J eV}^{-1}) = 1.77\ \text{eV}$$

The energy range is from 2.84×10^{-19} J to 4.97×10^{-19} J, or equivalently from 1.77 eV to 3.10 eV.

QUESTION 1.6

No. In the case of Figure 1.23 all of the sources being measured were *of the same size* and *at the same distance* from the detector. If sources of different sizes had been used, or if the sources had been at different distances from the detector, the results might well have been different. A 3000 K source of sufficient size or located sufficiently close to the detector could certainly provide more energy per second than a 6000 K source that was small or distant.

QUESTION 1.7

Assuming that the heated ball can be treated as a rough approximation to a black-body source of light, it is to be expected that the ball would emit relatively more light with wavelengths towards the blue end of the spectrum (as opposed to the red end) as the temperature increases, as indicated by Figure 1.23. Thus, at relatively low temperatures red light will predominate and the ball will glow ‘red-hot’. As the temperature rises the proportion of shorter wavelengths will gradually increase, making the colour progress from red to orange-white to yellowish-white to white – the last being a mix of all the colours.

QUESTION 1.8

It follows from Equation 1.1 that the wavelength of the absorbed electromagnetic radiation is

$$\lambda_{\mathrm{fi}} = \frac{v}{f_{\mathrm{fi}}}$$

However, the frequency of the electromagnetic wave is related to photon energy by Equation 1.5. Combining these two equations gives,

$$\lambda_{\mathrm{fi}} = \frac{vh}{E_{\mathrm{f}} - E_{\mathrm{i}}}$$

where v is the speed of light in the gas containing the atom. If the gas were very thin, then v would be approximately equal to the speed of light in a vacuum and we could write

$$\lambda_{\mathrm{fi}} = \frac{ch}{E_{\mathrm{f}} - E_{\mathrm{i}}}$$

QUESTION 1.9

On the whole, the chromosphere is very dim compared with the photosphere. Thus, we can expect to see chromospheric emissions at only those wavelengths where the photosphere is relatively dark. These will be the wavelengths at which the solar spectrum exhibits absorption lines, because most of the light at other wavelengths comes from the photosphere. In the case of the Hα line, for example, it is the chromosphere that is mainly responsible for the absorption of photospheric light at that wavelength, and it is also the chromosphere that is mainly responsible for the (weak) emitted light.

QUESTION 1.10

This question can be answered with the aid of the relative spectral flux densities (R) given in Figure 1.23. At 6000 K,

$$\frac{R_{6000}\ (400\ \mathrm{nm})}{R_{6000}\ (700\ \mathrm{nm})} = \frac{4.6}{4.0} = 1.2$$

At 5000 K the ratio would have been

$$\frac{R_{5000}\ (400\ \mathrm{nm})}{R_{5000}\ (700\ \mathrm{nm})} = \frac{1.3}{1.9} = 0.7$$

As you can see, the two ratios are very different. When treated properly, ratios of this kind are sufficient to determine the temperature of a black-body source. There is no need to record the entire spectrum or even to determine the precise location of the peak.

QUESTION 1.11

Wien's displacement law (Equation 1.4) needs to be rearranged to make (T/K) the subject,

$$(T/\mathrm{K}) = \frac{2.90 \times 10^{-3}}{(\lambda_{\mathrm{peak}}/\mathrm{m})}$$

The peak of the Sun's spectrum seems to be at a wavelength of about 470 nm. Substituting this value in Wien's displacement law gives

$$(T/\mathrm{K}) = \frac{2.90 \times 10^{-3}}{(470 \times 10^{-9}\ \mathrm{m}/\mathrm{m})} = 6170$$

$$T = 6.2 \times 10^3\ \mathrm{K}$$

This value is rather high, but we have taken a crude approach here. In fact, a Planck curve corresponding to a black-body source with a lower temperature (5800 K, say) provides a reasonable approximation to the data in Figure 1.30.

QUESTION 1.12

The completed Table 1.1 is shown below in Table 1.2.

Table 1.2 Completed version of Table 1.1.

Wavelength, λ/m	3×10^{-14}	6×10^{-10}	3×10^{-7}	1×10^{-5}	5×10^{-3}	10
Corresponding frequency, f/Hz	1×10^{22}	5×10^{17}	1×10^{15}	3×10^{13}	6×10^{10}	3×10^{7}
Corresponding photon energy, ε/J	7×10^{-12}	3×10^{-16}	6×10^{-19}	2×10^{-20}	4×10^{-23}	2×10^{-26}
Corresponding photon energy, ε/eV	4×10^{7}	2×10^{3}	4	1×10^{-1}	2×10^{-4}	1×10^{-7}
Temperature, T/K, of a black body that has a peak in its spectrum at this value of λ	1×10^{11}	5×10^{6}	9×10^{3}	3×10^{2}	1	3×10^{-4}
Corresponding part of the electromagnetic spectrum	γ-ray	X-ray	ultraviolet	infrared	microwave	radio wave

QUESTION 1.13

The only way to do this on the basis of the information given in this chapter is to compare the size of the plage in the calcium K image (Figure 1.19) with the size of the solar disc. This is similar to Question 1.2 and suffers from the same problem of foreshortening. At a rough estimate, the longest dimension of the plage is about a quarter of the diameter of the Sun, which implies a (maximum) size of

$$\tfrac{1}{4} \times 1.4 \times 10^6\ \mathrm{km} = 3.5 \times 10^5\ \mathrm{km}$$

QUESTION 1.14

No. The black-body spectrum depends only on the temperature of the source and not on any other physical characteristic such as its composition.

QUESTION 1.15

(a) It follows from Equation 1.5 that the frequency, f_{32}, of the radiation absorbed during a transition from E_2 to E_3 is $(1/h)(E_3 - E_2)$, which is

$$\frac{1}{4.14\times10^{-15}}\left[\left(-\frac{13.6}{3^2}\right)-\left(-\frac{13.6}{2^2}\right)\right]\text{Hz} = 4.56\times10^{14}\text{ Hz}$$

Note that because the photon energies are given in terms of electronvolts, the Planck constant must be expressed in terms of eV s, and so the value $h = 4.14 \times 10^{-15}$ eV s is used in the above calculation.

Now, the corresponding wavelength is given by

$$\lambda_{32} = \frac{c}{f_{32}} = \frac{3.00\times10^{8}}{4.56\times10^{14}}\text{ m} = 658\text{ nm}$$

which is close to the 656.3 nm measured for the Hα line.

Since the numerical work in this question has been carried out to three significant figures, the third figure in the final answer is actually somewhat suspect and it would be best to conclude that the given transition will absorb photons of wavelength between 650 nm and 660 nm. Clearly, such a conclusion makes it quite possible for the given transition to be responsible for Hα absorption, but it's hardly firm proof. What it does show is the need to work to much greater precision when dealing with spectroscopic quantities. In fact, more precise calculations *do* confirm that Hα absorption is due to the E_2 to E_3 transition.

(b) Hβ absorption is caused by the E_2 to E_4 transition (which can be discovered by trial and error). Thus

$$\begin{aligned}\lambda_{42} &= \frac{c}{f_{42}} = \frac{c}{(E_4 - E_2)/h} = \frac{ch}{(E_4 - E_2)}\\ &= \frac{3.00\times10^{8}\times4.14\times10^{-15}}{\left(-\frac{1}{4^2}+\frac{1}{2^2}\right)\times13.6}\text{ m} = 487\text{ nm}\end{aligned}$$

This is to be compared with the wavelength of Hβ absorption, which is 486.1 nm.

(c) Hβ emission is due to the E_4 to E_2 transition (the reverse of the transition causing Hβ absorption).

(d) The transition from E_1 to E_2 produces electromagnetic radiation with a wavelength that is only one-quarter of the Hβ wavelength. Such a wavelength is so small that it is outside the range covered by Figure 1.28. (It is actually in the ultraviolet part of the spectrum.)

QUESTION 1.16

(a) Ca^{14+} and Fe^{9+}.

(b) The energy carried by the photons that contribute to the yellow and red lines must arise from appropriate transitions in the atoms concerned. For the yellow line the relevant energy is

$$\begin{aligned}\varepsilon &= hf = h\frac{c}{\lambda}\\ &= \frac{(4.14\times10^{-15}\ \text{eV s})\times(3.00\times10^{8}\ \text{m s}^{-1})}{5.69\times10^{-7}\ \text{m}} = 2.18\,\text{eV}\end{aligned}$$

So this must represent the difference in energy between two of the energy levels of Ca^{14+}.

Similarly, in the case of the red line

$$\begin{aligned}\varepsilon &= \frac{(4.14\times10^{-15}\ \text{eV s})\times(3.00\times10^{8}\ \text{m s}^{-1})}{6.37\times10^{-7}\ \text{m}}\\ &= 1.95\,\text{eV}\end{aligned}$$

So this must represent the difference between two of the energy levels of Fe^{9+}.

QUESTION 1.17

(a) The approach here is the same as was taken in Example 1.1. The typical photon energy is found from Equation 1.6

$$\varepsilon \sim kT$$

Rather than inserting numerical values at this stage, it is better to find an expression for the wavelength. By combining Equations 1.2 and 1.3 as in Example 1.1, the wavelength is given by

$$\lambda = hc/\varepsilon$$

The photon energy is given by Equation 1.6, so we can write

$$\lambda \sim hc/kT$$

Notice that we have replaced the equals sign by '~' because this is now only an approximate relationship.

Now we can insert numerical values, taking care to ensure that the value of h is the appropriate one for the units we are using. The calculation is in SI units (we are not interested in expressing the energy in electronvolts here), so we must use the value $h = 6.63 \times 10^{-34}$ J s.

$$\begin{aligned}\lambda &\sim (6.63 \times 10^{-34}\ \text{J s}) \times (3.00 \times 10^{8}\ \text{m s}^{-1})/[(1.38 \times 10^{-23}\ \text{J K}^{-1}) \times (5 \times 10^{6}\ \text{K})]\\ &= 2.88 \times 10^{-9}\ \text{m}\end{aligned}$$

So the typical wavelength of emission from this source will be about 3 nm.

(b) If the source acts as a black body, the peak of the spectrum of emission occurs at a wavelength given by Wien's displacement law (Equation 1.4),

$$(\lambda_{\text{peak}}/\text{m}) = \frac{2.90\times10^{-3}}{(T/\text{K})} = \frac{2.90\times10^{-3}}{5\times10^{6}} = 5.8\times10^{-10}$$

So the peak of the black-body spectrum occurs at a wavelength of 6×10^{-10} m or 0.6 nm.

(c) At first sight it may appear that the wavelengths found in parts (a) and (b) are quite different, being 3 nm and 0.6 nm, respectively. However, the first calculation is an estimate of the wavelength that a thermal source may emit at, and is in fact, only expected to give an answer that is within a factor of 10 of the true value. Since these answers differ by a factor of 5 (= 3 nm/0.6 nm), the wavelengths calculated in parts (a) and (b) *are* consistent with one another.

QUESTION 1.18

At radio wavelengths, much of the Sun's emission comes from regions of the corona where temperature increases with height. Because (as argued in Section 1.2.2) observations made near the limb sample material that is, on average, at greater altitude than that sampled by observations made near the centre of the solar disc, it follows that at radio wavelengths limb observations will involve higher and hence hotter material than disc observations. Thus, at radio wavelengths the limb will be brighter than the disc centre.

At visible wavelengths the corona is essentially transparent and its own emissions are very faint. Thus there is no chance of observing limb brightening in visible light produced by the corona.

QUESTION 2.1

The volume occupied by the core of the Sun, V_c, is related to the radius of the core, R_c, by

$$V_c = \tfrac{4}{3}\pi R_c^3 \tag{i}$$

Similarly, the volume of the Sun, $V_\odot$, is related to the solar radius, $R_\odot$, by

$$V_\odot = \tfrac{4}{3}\pi R_\odot^3 \tag{ii}$$

To find an expression for the volume of the core as a fraction of the total volume of the Sun, Equation (i) is divided by Equation (ii)

$$\frac{V_c}{V_\odot} = \frac{\frac{4}{3}\pi R_c^3}{\frac{4}{3}\pi R_\odot^3}$$

The factor of $(4/3)\pi$ is common to both the denominator and the numerator and can so be cancelled to give

$$\frac{V_c}{V_\odot} = \frac{R_c^3}{R_\odot^3} = \left(\frac{R_c}{R_\odot}\right)^3$$

The question states that $R_c/R_\odot = 0.3$, so

$$\frac{V_c}{V_\odot} = (0.3)^3 = 2.7 \times 10^{-2}$$

So, to 1 significant figure, the volume of the core is 3% of the total volume of the Sun.

QUESTION 2.2

Since X departs significantly from a constant value only in the central 30% of the Sun (that is, where $R/R_\odot$ is less than 0.30) it seems pretty clear that the nuclear processes that convert hydrogen into helium must be confined to that inner region. Further, because X falls progressively as the fractional radius decreases in this inner region, conversion of hydrogen into helium must have been most common in the most central parts of the Sun.

QUESTION 2.3

(a) Following the initial reaction between two protons that was considered in Example 2.1, the second and third stages of the ppI chain are:

$$^{2}_{1}\text{H} + {}^{1}_{1}\text{H} \rightarrow {}^{3}_{2}\text{He} + \gamma \qquad (2)$$

$$2\,{}^{3}_{2}\text{He} \rightarrow {}^{4}_{2}\text{He} + 2\,{}^{1}_{1}\text{H} \qquad (3)$$

(b) For Reaction 2, the total incoming charge is $2e$ and so is the total outgoing charge.

For Reaction 3, the total incoming charge is that carried by the protons in *two* helium nuclei, so it is $4e$. The total outgoing charge is that carried by *one* helium nucleus and two hydrogen nuclei, $4e$.

Thus, in both reactions, electric charge is conserved.

(c) For both reactions the baryon number entering or leaving is equal to the total number of protons and neutrons entering or leaving, and this total is given by the superscript, the mass number. You can easily see that on both sides of Reaction 2 it is 3, and in Reaction 3 it is 6. The baryon number is thus conserved in both cases.

QUESTION 2.4

(a) The radiant energy eventually resulting from each occurrence of the ppI chain is given by the expression

$$c^2\,[\,(4 \times \text{mass of } {}^{1}_{1}\text{H}) - (\text{mass of } {}^{4}_{2}\text{He}) - (2 \times \text{mass of } \text{e}^{+}) + (2 \times \text{mass of } \text{e}^{+}) + (2 \times \text{mass of } \text{e}^{-})\,]$$

$$= 9.00 \times 10^{16}[4 \times 1.673 - 6.645 + 2 \times 0.001] \times 10^{-27}\,\text{J}$$
$$= 4.4 \times 10^{-12}\,\text{J}$$

The accurate value is about 5% lower than this estimate.

(b) If we assume (somewhat incorrectly) that the solar luminosity is entirely provided by the ppI chain and its supplementary reactions, then the number of times per second that the chain is completed is given by

$$\frac{3.84 \times 10^{26}\ \text{J s}^{-1}}{4.4 \times 10^{-12}\ \text{J}} = 8.7 \times 10^{37}\ \text{s}^{-1}$$

This is only an estimate, but it is not too bad.

(c) Each time the chain is completed, four hydrogen nuclei are consumed (and one helium nucleus produced). Since the mass of a hydrogen nucleus is 1.673×10^{-27} kg, it follows that the rate of hydrogen consumption is roughly

$$4 \times 8.7 \times 10^{37} \times 1.673 \times 10^{-27}\,\text{kg s}^{-1} = 5.8 \times 10^{11}\,\text{kg s}^{-1}$$

Now, the number of seconds in a year is 3.16×10^7. So the annual consumption of hydrogen is

$$3.16 \times 10^7 \times 5.8 \times 10^{11}\,\text{kg} = 1.8 \times 10^{19}\,\text{kg}$$

This is about three millionths of the Earth's mass per year, and about one part in 10^{11} of the Sun's mass per year.

QUESTION 2.5

(a) The wavelength of a photon of a given energy is found by combining Equations 1.2 and 1.3 and rearranging to give

$$\lambda = hc/\varepsilon$$

The energy of one of the γ-rays produced by electron–positron annihilation is 0.51 MeV or 0.51×10^6 eV. Since energies are being measured here in terms of eV rather joules, it is convenient to use $h = 4.14 \times 10^{-15}$ eV s. Hence

$$\lambda = \frac{4.14 \times 10^{-15} \times 3.00 \times 10^8}{5.1 \times 10^5}\,\text{m} = 2.4 \times 10^{-12}\,\text{m}$$

(b) The wavelength of the peak of the black-body curve is given by Equation 1.4,

$$(\lambda_{\text{peak}}/\text{m}) = \frac{2.90 \times 10^{-3}}{(T/\text{K})}$$

The temperature at the core of the Sun is approximately 1.5×10^7 K, so

$$\lambda_{\text{peak}} = \frac{2.90 \times 10^{-3}}{1.5 \times 10^7}\,\text{m} = 1.9 \times 10^{-10}\,\text{m}$$

According to Figure 1.36, this peak lies in the X-ray region of the electromagnetic spectrum.

QUESTION 2.6

Two neutrinos are released each time the ppI chain is executed (Section 2.2.4). Assuming (somewhat unrealistically) that all nuclear reactions apart from those of the ppI chain can be ignored, we know (from Question 2.4) that the ppI chain is completed about 8.71×10^{37} times per second. (This estimate involves various assumptions that are discussed in Question 2.4.) It follows that the rate at which neutrinos are produced in the core of the Sun is about $2 \times 8.71 \times 10^{37}\,\text{s}^{-1}$. Now, assuming (quite realistically) that all these neutrinos escape from the Sun and spread out evenly in all directions, the rate at which they pass through an area of $0.01\,\text{m}^2$ on a spherical surface of radius 1.50×10^{11} m, centred on the Sun, is

$$(2 \times 8.71 \times 10^{37}) \frac{0.01}{4\pi \times (1.50 \times 10^{11})^2}\,\text{s}^{-1} = 6.16 \times 10^{12}\,\text{s}^{-1}$$

Since $0.01\,\text{m}^2$ is roughly the cross-sectional area of a human brain, this calculation justifies the claim made earlier that more than a million million neutrinos pass through your head in the time it takes to read a sentence.

QUESTION 2.7

(a) To estimate the speed of the front of the CME, we need to measure the distance travelled by the front between two intervals. Distances on these images can be roughly estimated from the size of the solar disc that is indicated. The images taken at 19:42 and 21:18 on 5 August 1999 both show the front reasonably well. Between these two times, the front of the CME appears to move a distance (perpendicular to the line of sight) of about 2 solar diameters. The time elapsed is 1 hour 36 minutes, which is equivalent to 96 minutes or 5.760×10^3 s. So the speed is given by

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{2 \times 2 \times 6.96 \times 10^8\ \text{m}}{5.760 \times 10^3\ \text{s}} = 4.83 \times 10^5\ \text{m s}^{-1}$$

The distance travelled can only be roughly estimated from the images, so to an appropriate number of significant figures, the speed is $5 \times 10^5\ \text{m s}^{-1}$ or equivalently $500\ \text{km s}^{-1}$.

The key assumptions that are made here are (i) that the speed of the CME does not change dramatically between the times when the two images were taken, and (ii) that the CME is being ejected in a direction that is perpendicular to the line of sight.

(b) The time taken for the CME to reach the radius of the Earth's orbit from the Sun is

$$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{1.50 \times 10^{11}\ \text{m}}{5 \times 10^5\ \text{m s}^{-1}} = 3 \times 10^5\ \text{s}$$

So the CME will reach the radius of the Earth's orbit about 3×10^5 s or about 3.5 days after being ejected from the Sun.

The main assumption made here is that the speed of the CME remains roughly constant as it travels from the Sun to Earth.

A time delay of about 3 days is generally observed between the ejection of CMEs that are heading towards Earth and the onset of geomagnetic storms and auroral activity. So the estimate of the speed and travel time of a CME that has been calculated here corresponds well to the actual behaviour of CMEs.

QUESTION 2.8

The kinetic energy E_k of a body of mass m and speed v is given by

$$E_k = \tfrac{1}{2}mv^2$$

In this case, $m = 5 \times 10^{13}$ kg, and from the answer to Question 2.7(a)

$$v = 5 \times 10^5\ \text{m s}^{-1}$$

so $$E_k = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(5 \times 10^{13}) \times (5 \times 10^5)^2\ \text{J} = 6 \times 10^{24}\ \text{J}$$

So the order of magnitude estimate of the kinetic energy of a large CME is 10^{25} J.

QUESTION 2.9

The average distance from the Sun to Mercury is 5.79×10^{10} m, which can be expressed as $(5.79 \times 10^{10})/(1.50 \times 10^{11})$ AU = 0.386 AU.

The average distance from the Sun to Pluto is 5.90×10^{12} m, which can be expressed as $(5.90 \times 10^{12})/(1.50 \times 10^{11})$ AU = 39.3 AU.

The average distance from the Sun to Mercury can also be expressed in terms of $R_\odot$ as $(5.79 \times 10^{10})/(6.96 \times 10^{8})R_\odot = 83.2R_\odot$.

The average distance from the Sun to the Earth can be expressed in terms of $R_\odot$ as $(1.50 \times 10^{11})/(6.96 \times 10^{8})R_\odot = 216R_\odot$.

QUESTION 2.10

Consider an area of 1 m^2 at a distance from the Sun equal to the radius of the orbit of the Earth around the Sun. This imaginary area is set up so that it directly faces the Sun. By considering how far the solar wind will travel in 1 s after passing this surface, we can work out the volume of solar wind that passes through the surface in every second per 1 m^2. Since the speed of the solar wind is assumed to be 750 km s^{-1}, this volume is

$$1.00\,\mathrm{m}^2 \times 7.5 \times 10^5\,\mathrm{m} = 7.5 \times 10^5\,\mathrm{m}^3$$

The mass of material lost per 1 m^2 per second is this volume multiplied by the density of the solar wind near the Earth. (Recall that the mass of a proton is $m_\mathrm{p} = 1.67 \times 10^{-27}$ kg.)

$$\begin{aligned}\text{volume} \times \text{density} &= 7.5 \times 10^5\,\mathrm{m}^3 \times 7 \times 10^6\,\mathrm{m}^{-3} \times m_\mathrm{p} \\ &= 5.25 \times 10^{12} \times 1.67 \times 10^{-27}\,\mathrm{kg} = 8.77 \times 10^{-15}\,\mathrm{kg}\end{aligned}$$

This is a mass loss rate of 8.77×10^{-15} kg s^{-1} m^{-2}, where we have taken care to ensure that the units reflect the fact that this is the mass lost per unit time and per unit area. To find the total mass loss rate, this rate per unit area needs to be multiplied by the area of the sphere with radius equal to the radius of the Earth's orbit. The area of this sphere is

$$4\pi R^2 = 4 \times \pi \times (1.50 \times 10^{11})^2\,\mathrm{m}^2 = 2.83 \times 10^{23}\,\mathrm{m}^2$$

So the mass loss rate over this sphere is

$$2.83 \times 10^{23}\,\mathrm{m}^2 \times 8.77 \times 10^{-15}\,\mathrm{kg\,s^{-1}\,m^{-2}} = 2.48 \times 10^9\,\mathrm{kg\,s^{-1}}$$

In terms of solar masses per year this is a rate of

$$\frac{2.48 \times 10^9\,\mathrm{kg\,s^{-1}} \times (365 \times 24 \times 60 \times 60\,\mathrm{s})}{1.99 \times 10^{30}\,\mathrm{kg}} = 3.93 \times 10^{-14} M_\odot\,\mathrm{yr}^{-1}$$

So the mass loss rate due to the solar wind is, to 1 significant figure, 2×10^9 kg s^{-1} or $4 \times 10^{-14} M_\odot$ yr^{-1}.

QUESTION 2.11

The auroral emission arises from electronic transitions in neutral atoms or ions of those elements that are common in the Earth's atmosphere. (Note also that electronic transitions in molecules, such as N_2 are also possible.) At the Earth's surface the atmosphere comprises 78% molecular nitrogen and 20% molecular oxygen. In the high-energy environment of the upper atmosphere it is likely that molecular oxygen and nitrogen could be split up into atomic form and possibly ionized. Hence it is likely that the aurora would show spectral lines due to atomic oxygen (a) and ionized nitrogen (c). There is a negligible amount of iron in the Earth's atmosphere, so a spectral line due to atomic iron (b) is unlikely to be seen.

QUESTION 2.12

The speed of the fast component of the solar wind is $750\,\mathrm{km\,s^{-1}}$, and the distance from the Sun to the termination shock is estimated to be 85 AU.

The travel time t, is given by

$$t = \frac{\text{distance}}{\text{speed}}$$

$$\text{distance} = 85\,\mathrm{AU} = 85 \times 1.50 \times 10^{11}\ \mathrm{m} = 1.275 \times 10^{13}\ \mathrm{m}$$

$$\text{speed} = 750\,\mathrm{km\,s^{-1}} = 7.50 \times 10^{5}\ \mathrm{m\,s^{-1}}$$

$$t = \frac{1.275 \times 10^{13}\ \mathrm{m}}{7.50 \times 10^{5}\ \mathrm{m\,s^{-1}}} = 1.7 \times 10^{7}\ \mathrm{s}$$

In terms of days this is

$$t = \frac{1.7 \times 10^{7}\ \mathrm{s}}{60 \times 60 \times 24\,\mathrm{s}} = 1.97 \times 10^{2}\ \text{days}$$

So the travel time from the Sun to the termination shock is about 200 days.

QUESTION 2.13

The composition of the Sun is roughly 73% hydrogen and 25% helium, by mass. Thus, for every 73 kg of hydrogen in the Sun there are 25 kg of helium and 2 kg of everything else. Since the mass of a hydrogen nucleus, ${}^{1}_{1}\mathrm{H}$, is 1.673×10^{-27} kg and that of a helium nucleus, ${}^{4}_{2}\mathrm{He}$, is 6.645×10^{-27} kg (these figures were given in Question 2.4), it follows that 100 kg of typical solar material will contain roughly

$$\frac{73}{1.673 \times 10^{-27}} = 4.36 \times 10^{28}\ \text{hydrogen nuclei}$$

and $$\frac{25}{6.645 \times 10^{-27}} = 3.76 \times 10^{27}\ \text{helium nuclei}$$

This is a ratio by *number* of nuclei of roughly 100 : 8.6, hydrogen to helium.

Now, the total mass of the Sun was given in Section 2.2.4 as 1.99×10^{30} kg. Thus, the total number of hydrogen nuclei in the Sun will be roughly

$$\frac{1.99 \times 10^{30}}{100} \times 4.36 \times 10^{28} = 8.7 \times 10^{56}$$

and the number of helium nuclei will be roughly

$$\frac{1.99 \times 10^{30}}{100} \times 3.76 \times 10^{27} = 7.5 \times 10^{55}$$

QUESTION 2.14

According to Question 2.4, the mass of hydrogen consumed every year is about 1.84×10^{19} kg. Since the total mass of the Sun is 1.99×10^{30} kg, and about 73% of this is hydrogen, it follows that the maximum duration of the conversion of hydrogen into helium at the present rate is

$$\frac{0.73 \times 1.99 \times 10^{30}}{1.84 \times 10^{19}} \text{ years} = 7.9 \times 10^{10} \text{ years}$$

Of course, this is a crude estimate because it assumes that all of the hydrogen currently in the Sun will undergo conversion and it also makes use of a result (from Question 2.4) that is itself only an estimate. Nonetheless, the final answer is quite reasonable. More refined estimates of the Sun's hydrogen-fuelled lifetime provide figures of about 10^{10} years. Since the Sun is currently thought to be about 4.5×10^{9} years old, it is usual to regard the Sun as a middle-aged star.

QUESTION 2.15

(a) The nucleus ${}^{5}_{3}\text{He}$ does not exist. Every helium nucleus *must* have an atomic number of 2, denoting two protons in the nucleus. The atomic number of 3 corresponds to the element lithium.

(b) This reaction does not conserve electric charge.

(c) This reaction also fails to conserve electric charge.

QUESTION 2.16

The solar luminosity was given earlier as $3.84 \times 10^{26}\ \text{J s}^{-1}$. Since there are approximately 3.16×10^{7} seconds in one year, it follows that the annual energy output of the Sun, E, is about 1.21×10^{34} J. Assuming that this energy is entirely supplied by the loss of mass from the Sun's core, it follows that the mass lost per year is

$$m = \frac{E}{c^2} = \frac{1.21 \times 10^{34}\ \text{J}}{(3.00 \times 10^{8}\ \text{m s}^{-1})^2} = 1.34 \times 10^{17}\ \text{kg}$$

It is worth emphasizing that this means the Sun is losing mass at the amazing rate of 4.25×10^{9} kilograms per second, and will lose about 10^{27} kg in its hydrogen-burning lifetime, which is about 0.05% of its mass.

QUESTION 2.17

From Question 2.4(a) the energy generated in a single ppI reaction is approximately 4×10^{-12} J. Given that this question asks for estimated values, it is appropriate to use a value given to 1 significant figure.

To estimate the number of photospheric photons that eventually carry this energy away, it is necessary to calculate the average energy of a photon that is emitted by the photosphere. A reasonable estimate of this can be made by assuming that the average photon energy corresponds to the peak of the Planck curve at the photospheric temperature. Using Equation 1.4 and a photospheric temperature of 6×10^3 K (i.e. to 1 significant figure),

$$(\lambda_{\text{peak}}/\text{m}) = \frac{2.90 \times 10^{-3}}{(T/\text{K})} = \frac{2.90 \times 10^{-3}}{6000} = 4.8 \times 10^{-7}$$

The corresponding photon energy is found by combining Equations 1.2 and 1.3

$$\varepsilon = hc/\lambda$$

$$= 6.6 \times 10^{-34}\ \text{J s} \times 3.0 \times 10^{8}\ \text{m s}^{-1}/4.8 \times 10^{-7}\ \text{m} = 4.1 \times 10^{-19}\ \text{J}$$

(Note that energies are given in joules, so the appropriate value of Planck's constant is $h = 6.6 \times 10^{-34}$ J s.)

So the number of these photons that would be required to carry away the energy released in a single ppI reaction is

$$\text{number of photons} = 4 \times 10^{-12}\ \text{J}/4.1 \times 10^{-19}\ \text{J} = 9.8 \times 10^{6}$$

So, to 1 significant figure, the number of photospheric photons that carry away the energy generated by a single ppI reaction is 1×10^7.

QUESTION 2.18

The field pattern that is proposed in Figure 2.40 is a loop which is modified by the fact that at the highest part of the loop there is a small dip in the magnetic field lines. Plasma can only move along field lines, so any plasma that lies within the dip will move under the influence of gravity to the lowest point of the dip. It is only in this position that the field lines can provide stable support against gravity. Any plasma that lies outside of the dip would also move along the field lines because of the gravitational force acting on it, but in this case it would not be supported – it would simply follow the field lines down to the chromosphere.

QUESTION 2.19

Energy that is stored in the magnetic field is converted into the thermal energy of the plasma at the site of reconnection and into the kinetic energy of particles in the plasma as field lines move away from this site.

QUESTION 3.1

With a parallax of 0.287 arcsec, the distance to 61 Cygni in parsecs is given by Equation 3.7 as

$$d/\text{pc} = 1/0.287 = 3.48$$

Thus $d = 3.48$ pc. This is $3.48 \times 206\,265$ AU $= 7.2 \times 10^5$ AU, and $7.2 \times 10^5 \times 1.50 \times 10^{11}$ m $= 1.1 \times 10^{17}$ m. The distance to the Sun is 1 AU, so 61 Cygni is 720 000 times further away.

QUESTION 3.2

The angular diameter of Betelgeuse is given in Section 3.3.1 as 0.050 arcsec, which is 0.050/206 265 = 2.42×10^{-7} radians. Thus, from Equation 3.8, the radius of Betelgeuse is given by

$$R = [(\alpha/2)/\text{radians}] \times d$$
$$= (2.42 \times 10^{-7}/2) \times 131\ \text{pc} = 1.59 \times 10^{-5}\ \text{pc} = 4.91 \times 10^{11}\ \text{m}$$

So the radius is 4.9×10^{11} m (to 2 significant figures).

$$1 \text{ solar radius} = 6.96 \times 10^{8}\ \text{m, so } R = 706R_{\odot}.$$

So the radius of Betelgeuse is about 700 times that of the Sun.

QUESTION 3.3

From Figure 3.23 we see that the Balmer lines are weak at temperatures above approximately 40 000 K and below about 5000 K. The corresponding spectral classes (Table 3.2) are O, K and M (G is a marginal case). Figures 3.25 and 3.26 confirm that Balmer lines are strong in spectral types B, A, F and G, but weak in the other spectral classes.

QUESTION 3.4

From Figure 1.38, the more transparent bands in the Earth's atmosphere are: the visible region; some bands in the near infrared region; most of the microwave and radio wave regions.

QUESTION 3.5

(a) Using your calculator, $\log 5 = 0.699$.

For the remaining parts of this question you do not need a calculator.

(b) 50 is 10 times larger than 5, so $\log 50 = (\log 5) + 1 = 1.699$

(c) 5000 000 is 10^6 times larger than 5, so $\log 5000\,000 = (\log 5) + 6 = 6.699$

(d) 0.5 is 10 times *smaller* than 5, so $\log 0.5 = (\log 5) - 1 = -0.301$

(e) 5×10^{-7} is 10^{-7} times smaller than 5, so $\log(5 \times 10^{-7}) = (\log 5) - 7 = -6.301$.

QUESTION 3.6

Using Equation 3.15, with star 1 as α Centauri A:

$$(0 - m(\alpha\ \text{Cen B})) = -2.5 \log(2.5), \text{ so } m(\alpha\ \text{Cen B}) = 1.0$$

$$(0 - m(\alpha\ \text{Cen C})) = -2.5 \log(25\,000), \text{ so } m(\alpha\ \text{Cen C}) = 11.0$$

You may have noticed that in this case it isn't necessary to use Equation 3.15 as you have learnt that a ratio of approximately 2.5 in flux density corresponds to a magnitude difference of 1. (More precisely, a ratio of 2.512 = 1 magnitude, which is not to be confused with the factor of 2.5 in Equation 3.15.) Also, a ratio of 25 000 is $2.5 \times 100 \times 100$, i.e. a magnitude difference of approximately $1 + 5 + 5 = 11$.

QUESTION 3.7

Using Equation 3.16, with magnitudes in the visual band,

$$M_V = m_V - 5 \log d + 5$$

For α Centauri: $M_V = 0.01 - 5 \log(1.35) + 5 = 4.36$

For β Centauri: $M_V = 0.61 - 5 \log(161) + 5 = -5.42$

α Centauri is of similar absolute visual magnitude to the Sun ($M_V = 4.8$) and therefore similar luminosity. β Centauri has a numerically much smaller absolute visual magnitude and is therefore much more luminous than the Sun.

QUESTION 3.8

From Equation 3.14

$$d = [6.1 \times 10^{30}\ \mathrm{W}/(4\pi \times 4.4 \times 10^{-10}\ \mathrm{W\ m^{-2}})]^{1/2}$$

$$= 3.32 \times 10^{19}\ \mathrm{m}$$

$$= 1070\ \mathrm{pc}$$

So, to 2 significant figures, the distance to ι^1 Sco is 1100 pc.

The value of L_V for ι^1 Sco is 1.4×10^5 times that of the Sun (4.44×10^{25} W), so its intrinsic visual brightness is very high. It is thus its distance that makes it seem not very bright. *Note*: ι^1 Sco is in fact both hotter and bigger than the Sun.

QUESTION 3.9

In obtaining temperature, we compare the strengths of absorption lines from *different* elements, whereas in obtaining composition we compare the strengths of different absorption lines from a *single* element.

QUESTION 3.10

From Appendix A5, the ten most abundant elements in the material from which the Solar System was formed are as follows, in descending order of abundance:

Order	By number of nuclei	By mass
1	hydrogen	hydrogen
2	helium	helium
3	oxygen	oxygen
4	carbon	carbon
5	neon	neon
6	nitrogen	iron
7	magnesium	nitrogen
8	silicon	silicon
9	iron	magnesium
10	sulfur	sulfur

QUESTION 3.11

Squaring both sides of Equation 3.18 we get

$$P^2 = \frac{4\pi^2 r^3}{G(M+m)}$$

We then divide both sides by P^2, and multiply both sides by $(M + m)$, to obtain the desired result:

$$M + m = \frac{4\pi^2 r^3}{GP^2}$$

QUESTION 3.12

If $M/m = 3$, $M = 3m$. Therefore we can substitute $3m$ for M into the equation for the sum of the masses, i.e. $M + m = 12M_\odot$, to obtain

$$(3m + m) = 12M_\odot$$

so $$m = 3M_\odot$$

Therefore, with $M/m = 3$,

$$M = 9M_\odot$$

QUESTION 3.13

(a) The angular diameter of the Moon (0.5°) is 1800 arcsec. So, at the rate of 1.259 arcsec yr^{-1}, it will take Procyon A 1800/1.259 = 1430 years to travel this angular distance.

(b) The transverse speed is obtained from the proper motion by means of Equation 3.1. Thus we must first obtain the distance d to Procyon. This is obtained using Equation 3.7:

$$d/\text{pc} = 1/0.286$$

Thus $$d = 3.50\,\text{pc} = 1.08 \times 10^{14}\,\text{km}$$

Secondly, the proper motion is given as 1.259 arcsec yr^{-1}. There are 206 265 arcsec in a radian (Section 3.2.2), and 3.16×10^7 seconds in a year, so this proper motion corresponds to

$$\left(\frac{1.259}{206\,265}\right)\left(\frac{1}{3.16 \times 10^7}\right) \text{radians per second}$$

that is 1.93×10^{-13} radians s^{-1}. Thus, from Equation 3.1,

$$v_\text{t} = d \times (\mu/\text{radians})$$
$$= (1.08 \times 10^{14}\,\text{km}) \times (1.93 \times 10^{-13}\,\text{s}^{-1}) = 20.8\,\text{km s}^{-1}$$

(c) Its spectral lines are blue-shifted, so its radial velocity is directed towards us.

(d) The large proper motion suggests that Procyon A is a nearby star, since from Equation 3.1 we see that nearby stars will on average have large proper motions. If Procyon A is nearby then it will have a large parallax.

QUESTION 3.14

(a) Using Equation 3.15, set star 1 as Sirius A and star 2 as Sirius B.

Then $b_1/b_2 = 7600/1$

and $(-1.46 - m_2) = -2.5\log(7600)$

so V magnitude of Sirius B is $m_2 = -1.46 + 2.5\log(7600) = 8.24$

(b) Since both stars are at the same distance, the difference in absolute visual magnitudes is the same as the difference in apparent visual magnitudes ($m_2 - m_1 = 9.70$).

(c) Using Equation 3.16, the absolute visual magnitude of Sirius A is given by

$$M_V = m_V - 5\log d + 5 = -1.46 - 5\log(2.64) + 5 = 1.43$$

(d) The Sun's absolute visual magnitude is 4.8. Sirius A has a numerically smaller absolute visual magnitude so its intrinsic brightness and hence its luminosity is higher than the Sun's.

QUESTION 3.15

(a) From Table 3.2 we see that 13 000 K is a reasonable temperature for a B8 star. At 13 000 K, the Balmer lines are strong compared with the helium lines, and very much stronger than the lines of ionized helium and ionized calcium (Figure 3.23). Remember that the actual procedure runs the other way: from the observed line strengths we establish the spectral type.

(b) A far less luminous star of the same temperature would have broader spectral lines, and weaker lines from certain ionized atoms.

(c) Following the method in Example 3.2, we get

$$R/R_\odot \approx (1.4 \times 10^5)^{1/2} \times (5770/13\,000)^2 \approx 74$$

Therefore $R = 74R_\odot$.

(d) From Figures 3.18 and 3.28, a rough estimate is that about 10% of its luminosity lies in the V waveband. Any value between about 5% and 15% is reasonable. Thus, with $L = 1.4 \times 10^5 L_\odot$, we get

$$L_V \approx 0.1 \times 1.4 \times 10^5 \times 3.84 \times 10^{26}\,\text{W} = 5.4 \times 10^{30}\,\text{W}$$

Thus, the approximate value of L_V is 5×10^{30} W.

To find the distance we use Equation 3.14

$$d \approx [5.4 \times 10^{30}\,\text{W}/(4\pi \times 3.0 \times 10^{-9}\,\text{W m}^{-2})]^{1/2}$$
$$= 1.20 \times 10^{19}\,\text{m} = 388\,\text{pc}$$

Thus the estimated distance to Rigel is $d \sim 400$ pc.

Note that the actual distance to Rigel A is about 240 pc.

(e) The angular diameter of Rigel A, from the Earth, is given by a straightforward rearrangement of Equation 3.8. Thus, using the values of radius R and distance d obtained earlier in this question, we get

$$\alpha = (2 \times 74 \times 6.96 \times 10^8 \text{ m})/(1.2 \times 10^{19} \text{ m}) \text{ radians}$$
$$= 8.6 \times 10^{-9} \text{ radians}$$
$$= 8.6 \times 10^{-9} \times 206\,265 \text{ arcsec}$$
$$= 0.002 \text{ arcsec}$$

This is measurable: at present the smallest measured values are about 0.0004 arcsec (Section 3.3.1).

QUESTION 3.16

From Appendix A5 we obtain Figure 3.42 for the heavy elements ($Z > 2$) as far as $Z = 30$. There is no simple trend with Z. Instead, there are some notably large values, as follows (values of Z in parentheses): carbon (6), nitrogen (7), oxygen (8), neon (10), magnesium (12), silicon (14), sulfur (16) and iron (26).

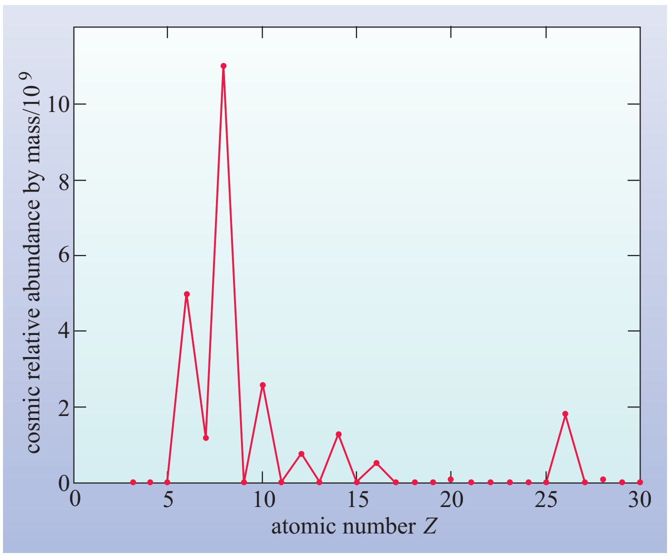


Figure 3.42 Elemental abundances in the Solar System by mass, with respect to H = 10^{12}, for the elements $2 < Z < 31$.

The reasons for this relationship with Z will be clearer to you by the end of the book.

QUESTION 3.17

From Equation 3.20 the sum of masses is

$$M + m = 4\pi^2 \frac{(20 \times 1.50 \times 10^{11} \text{ m})^3}{(6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}) \times (50 \times 3.16 \times 10^7 \text{ s})^2}$$
$$= 6.4 \times 10^{30} \text{ kg}$$

From Figure 3.37

$$d_m/d_M = 2.3$$

This ratio is not influenced by our projected view of the orbit.

Thus, from Equation 3.21

$$M/m = 2.3$$

We thus have

$$(2.3m + m) = 6.4 \times 10^{30}\,\text{kg}$$

and so $m = 1.9 \times 10^{30}\,\text{kg} = 1.0M_\odot$

Thus $M = 2.3 \times 1.0M_\odot = 2.3M_\odot$

Note also that if we measure P in years, and a in AU, then we can write Equation 3.20 as

$$(M + m)/M_\odot = (a/\text{AU})^3/(P/\text{yr})^2$$

This numerical simplification arises from the Earth–Sun system, for which $M = M_\odot$, $M_\odot >> M_E$ (Earth's mass), $P = 1$ year and $a = 1$ AU. You may want to try answering this question using this version of Equation 3.20.

QUESTION 4.1

The positions in the H–R diagram of these types of star are as follows:

hot, high luminosity stars	top left
hot, low luminosity stars	bottom left
cool, low luminosity stars	bottom right
cool, high luminosity stars	top right

QUESTION 4.2

From Figure 4.5, we see that white dwarfs have radii of order $0.01R_\odot$, which is about the radius of the Earth. Likewise, we see that red giants have radii of order $30R_\odot$, which is about 3000 times the Earth's radius, or about a tenth of the distance of the Earth from the Sun. Note that the *ranges* of radii for white dwarfs, and particularly for red giants, are large.

QUESTION 4.3

Such stars would move diagonally to the right and downwards, the luminosity as well as the temperature decreasing. You will see later that many stars do indeed end their lives in this way.

QUESTION 4.4

(a) The H–R diagram of M67 in Figure 4.10b is notable for the absence from the main sequence of all but the low-mass stars (Figure 4.8), and the presence of considerable numbers of stars between the main sequence and the red giant region, which could represent the higher masses missing from the main sequence. This suggests that the more massive a star, the sooner it leaves the main sequence, and that most stars that have left the main sequence go on to become red giants. Supergiants are absent in M67, and this could be because massive main sequence stars, which are their precursors, are rare. Also, if, as it seems, massive stars evolve rapidly, then any supergiants could have become Type II supernovae, and

have thus vanished from the H–R diagram. The absence of white dwarfs is presumably because they are too faint to detect. Thus, the H–R diagram for M67 is consistent with the model of stellar evolution in Figure 4.12.

(b) Assuming the model is right, we can conclude that M67 is older than the Pleiades, because in the Pleiades the main sequence is populated to higher stellar masses than the main sequence in M67 (Figure 4.10). This occurs because the more massive the star the sooner it leaves the main sequence. In M67 there has been enough time for all but the low mass stars to leave the main sequence, whereas the Pleiades is too young for this to have happened.

QUESTION 4.5

From Figure 4.16 we see that, for CO, the difference in energy ε between the lowest electronic level and the one above it is 5.94 eV ($= 9.52 \times 10^{-19}$ J).

ε corresponds to a photon wavelength given by

$$\lambda = hc/\varepsilon$$
$$= (6.63 \times 10^{-34}\,\text{J s}) \times (3.00 \times 10^{8}\,\text{m s}^{-1})/(9.52 \times 10^{-19}\,\text{J})$$
$$= 2.09 \times 10^{-7}\,\text{m} = 209\,\text{nm}$$

This is the maximum photon wavelength (minimum energy) for this excitation.

(b) From Equation 4.3, the minimum temperature is given by

$$T = 2\varepsilon/(3k)$$
$$= 2 \times 9.52 \times 10^{-19}\,\text{J}/(3 \times 1.38 \times 10^{-23}\,\text{J K}^{-1}) = 4.60 \times 10^{4}\,\text{K}$$

So typically a temperature of at least 5×10^4 K is required for the collisional excitation of this electronic state in CO molecules.

QUESTION 4.6

Equation 3.16 does not take account of interstellar extinction, A, as in Equation 4.1:

$$M = m - 5 \log d + 5 - A$$

The derived absolute visual magnitude will therefore be too faint (M numerically too large). Since interstellar dust also causes reddening, the $B - V$ colour will be redder and therefore the derived temperature will be too low. Examination of the axes of the H–R diagram in Figure 4.5 shows that the star will appear below and to the right of its correct position.

If a spectrum is observed, the temperature can be derived from its spectral type (based on the strengths of certain spectral lines, Section 3.3.2) and therefore not affected by interstellar reddening. Its luminosity can also be inferred directly from its spectrum (Section 3.3.4) and hence its true position on the H–R diagram can be determined.

QUESTION 4.7

The distance to a star does not influence its position on the H–R diagram: photospheric temperature and luminosity are intrinsic properties of a star. However, we sometimes have to apply corrections to our observations, because of interstellar extinction, in order to obtain intrinsic properties.

QUESTION 4.8

(a) Figure 4.22 shows the five stars plotted on an H–R diagram. By comparison with Figure 4.5, we can make the following assignments given in Table 4.2.

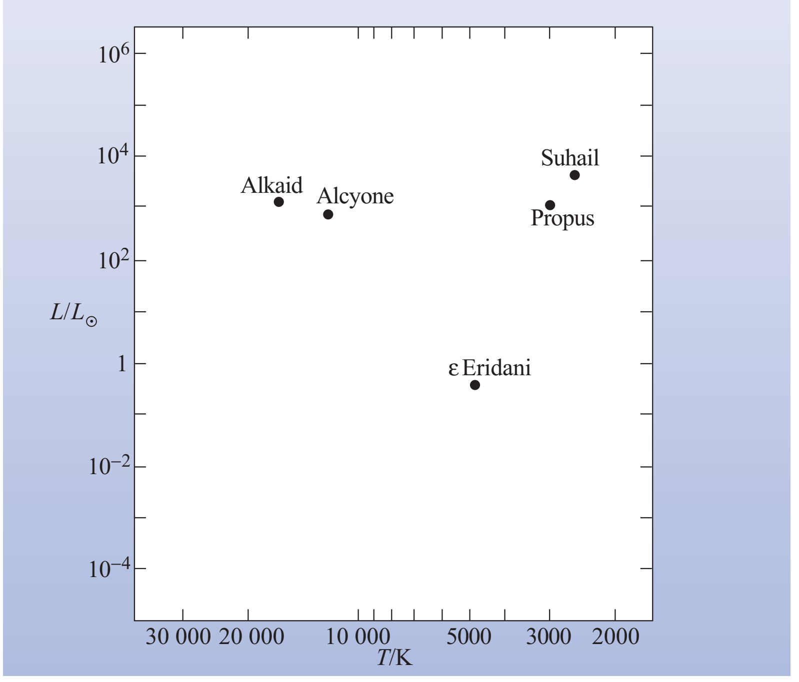


Figure 4.22 An H–R diagram with five stars, from Question 4.8.

Table 4.2

Star	Main stellar class	Comment
Alkaid	main sequence	
Alcyone	giant	
ε Eridani	main sequence	a nearby (3.2 pc), solar-type star
Propus	red giant	
Suhail	red giant/supergiant	some stars fall between the four main classes

(b) Low luminosity stars can have a large apparent visual brightness only if they are particularly close to us. Thus, low-luminosity stars will be under-represented in an H–R diagram that contains only the stars with the greatest apparent visual brightness. (There is always a likelihood that low-luminosity stars will be under-represented, simply because any detection system will fail to detect stars that give too low a flux density at the detector. This is an example of a *selection effect*.)

QUESTION 4.9

From Figure 4.5 we see that the Sunís photospheric temperature, $T_\odot$, luminosity, $L_\odot$, and radius, $R_\odot$, have the following relationships to the temperature T, luminosity L and radius R of stars near the upper and lower ends of the main sequence:

upper end: $T \sim 6T_\odot$, $L \sim 10^6 L_\odot$, $R \sim 10R_\odot$

lower end: $T \sim 0.3T_\odot$, $L \sim 10^{-4} L_\odot$, $R \sim 0.03R_\odot$

Thus, the Sun is a very modest main sequence star.

QUESTION 4.10

The region where T Tauri stars are found on the HñR diagram is shown in Figure 4.7. Since the question states that T Tauri stars approach the main sequence with little change in temperature, they must follow vertical paths from this region to the main sequence. By inspection of Figure 4.5 it can be seen that by following a vertical track from the T Tauri region to the main sequence a small reduction in radius must occur.

QUESTION 4.11

To evolve into a supergiant, the red giant would have to acquire a lot of mass: see Figure 4.8. However, observations suggest that red giants *lose* mass (in the form of stellar winds), and so we can rule out the scenario in which a red giant evolves into a supergiant.

QUESTION 5.1

Note: We choose nitrogen to represent the whole of the Earthís atmosphere as it is the most abundant gas (its mass is very similar to that of oxygen, which is the other major component).

The common isotope of nitrogen, ${}^{14}_{7}\mathrm{N}$, has an atomic mass of $14 \times (1.67 \times 10^{-27}\,\mathrm{kg})$. Nitrogen is present in its molecular form in the Earthís atmosphere so the number n of molecules per cubic metre is given by

$$n \approx \frac{1\,\mathrm{kg\,m^{-3}}}{14 \times 2 \times (1.67 \times 10^{-27}\,\mathrm{kg})} \approx 2 \times 10^{25}\,\mathrm{m^{-3}}$$

Comparison with Figure 5.1 shows that the atmosphere at the surface of the Earth is about 10^9 times more dense than the densest part of the ISM.

The surface temperature, about 300 K, is not much greater than that of the warmer diffuse clouds.

QUESTION 5.2

For an object that is emitting as a black-body source at temperature T, the wavelength, λ_{peak}, at which the maximum energy is radiated is given by Wienís displacement law, (Equation 1.4):

$$\left(\lambda_{\mathrm{peak}}/\mathrm{m}\right) = \frac{2.90 \times 10^{-3}}{(T/\mathrm{K})}$$

For a dense cloud collapsing on its way to becoming a protostar, we expect the temperature to still be very low, probably still only a few hundred kelvin – let us assume 300 K. In this case

$$\lambda_{\text{peak}} = \frac{2.90 \times 10^{-3}\ \text{m}}{300} \approx 10^{-5}\ \text{m}$$

This is in the infrared part of the spectrum (see Figure 1.36).

If we had assumed a temperature different by a factor of 10, larger or smaller, our answer would still have been in the infrared!

QUESTION 5.3

The smaller the Jeans mass, the more likely gravitational contraction is to occur. From Equation 5.1, we can see that a small value for the Jeans mass requires that n, the number density, be as large, and T, the temperature, be as low, as possible. The same conclusion could be derived from Figure 5.9.

QUESTION 5.4

The number of stars observed in a given phase of evolution should be roughly proportional to the time an individual star spends in that phase. When evolution is rapid, the phase of evolution is soon over and the probability of our observing objects in that phase is low. (One exception to this is a supernova – but that will be discussed later.)

QUESTION 5.5

In bipolar outflow, the flow is highly directed. This means that observations of the Doppler shift will tend to show two distinct values (except for the case where the outflow is perpendicular to the line of sight) corresponding to flow in two opposite directions. For a T Tauri star, outflow is in all directions, so a range of Doppler shifts can be observed. In addition, the outflow from T Tauri stars tends to have a higher velocity, leading to a larger Doppler shift. Furthermore, T Tauri stars sometimes show variable outflow.

QUESTION 5.6

The mass of a hydrogen molecule is $2 \times 1.67 \times 10^{-27}$ kg.

Thus the total mass of the cloud is

$$\begin{aligned}\text{mass} &= \text{volume} \times \text{density}\\ &= [\tfrac{4}{3}\pi \times (3 \times 3.09 \times 10^{16}\ \text{m})^3] \times [10^9\ \text{m}^{-3} \times (2 \times 1.67 \times 10^{-27}\ \text{kg})]\\ &= 1.1 \times 10^{34}\ \text{kg}\end{aligned}$$

If we assume, for the present calculation, that all stars have a mass equal to that of the Sun (2×10^{30} kg), we can work out how many stars could be formed from the cloud.

$$\text{number of stars} = (1.1 \times 10^{34}\ \text{kg})/(2 \times 10^{30}\ \text{kg}) \approx 5.5 \times 10^3$$

If we assume therefore that all the material in the original cloud goes into making stars, we find that there is plenty of material to make a large number of stars.

In Chapter 3, we learnt that stars generally have masses in the range from about $0.08M_\odot$ to about $50M_\odot$, so our conclusion wouldn't change whatever mass we assumed for the stars that formed from this cloud.

QUESTION 5.7

We calculate the Jeans mass in each case using Equation 5.1:

$$M_{\rm J} = \frac{9}{4} \times \left(\frac{1}{2\pi n}\right)^{1/2} \times \frac{1}{m^2} \times \left(\frac{kT}{G}\right)^{3/2}$$

Using the parameters for the two cases from the question and $m = 2 \times 1.67 \times 10^{-27}$ kg for the mass of the hydrogen molecule, we find, in the uncompressed case

$$M_{\rm J} = \frac{9}{4}\left[\frac{1}{2\pi \times 5 \times 10^9\ \mathrm{m}^{-3}}\right]^{1/2} \times \frac{1}{(3.34 \times 10^{-27}\ \mathrm{kg})^2} \times \left[\frac{1.38 \times 10^{-23}\ \mathrm{J\,K}^{-1} \times 10\,\mathrm{K}}{6.67 \times 10^{-11}\ \mathrm{N\,m}^2\,\mathrm{kg}^{-2}}\right]^{3/2}$$

$$= \tfrac{9}{4} \times (5.6 \times 10^{-6}\,\mathrm{m}^{3/2}) \times (8.9 \times 10^{52}\,\mathrm{kg}^{-2}) \times (3.0 \times 10^{-18}\,\mathrm{m}^{-3/2}\,\mathrm{kg}^3)$$

$$= 3.4 \times 10^{30}\,\mathrm{kg}$$

Thus, in solar masses ($M_\odot = 2 \times 10^{30}$ kg) (and rounding to 1 significant figure, consistent with the information given in the question) $M_{\rm J} = 2M_\odot$.

Therefore, the core, mass $M_\odot$, has a mass *less* than its Jeans mass, so it is unlikely to contract, particularly if rotation or magnetic fields hinder contraction. In the compressed case, a similar calculation yields $M_{\rm J} = 0.5M_\odot$.

Thus, the core mass is now *greater* than its Jeans mass, so it is likely to contract unless supported by rotation or magnetic fields.

QUESTION 6.1

The pressure is assumed to be uniform over the whole area of contact between the two hemispheres. In reality it will vary, being highest in the centre.

The two hemispheres are assumed to be point masses. The mass is distributed throughout the hemispheres which are very close together.

(In real stellar models the material is treated as a series of thin concentric spherical shells.)

QUESTION 6.2

The composition of the core could change during the main sequence through (i) convection and (ii) fusion reactions. Figure 6.5 shows that, in stars of low mass, convection is confined to a thin outer envelope, so convective mixing of core material with the material around it won't occur. Therefore the only way in which the composition of the core will change during the main sequence lifetime is through fusion reactions. As in the Sun, these result in a progressive depletion of hydrogen and enrichment of helium in the core.

QUESTION 6.3

The time for which the Sun's present luminosity could be maintained is given by (energy available)/(rate of energy radiation), where the rate of energy radiated is the Sun's luminosity. The energy available from chemical energy is given by the Sun's mass multiplied by the energy available per unit mass:

$$(2 \times 10^{30}\,\text{kg}) \times (3.5 \times 10^{7}\,\text{J kg}^{-1}) = 7 \times 10^{37}\,\text{J}$$

Thus, the time for which the Sun's present luminosity (of 4×10^{26} W) could be maintained is

$$(7 \times 10^{37}\,\text{J})/(4 \times 10^{26}\,\text{W}) = 2 \times 10^{11}\,\text{s, i.e. } {\sim}6000 \text{ years!}$$

Evidence shows that the Sun has been shining at (approximately) its present luminosity for about 5×10^{9} years. Therefore the generation of energy by chemical means does not appear to be the likely explanation of the Sun's luminosity.

QUESTION 6.4

The loss of gravitational energy is given by

$$\begin{aligned}
\frac{-GM_\odot^2}{R_\odot} - \frac{-GM_\odot^2}{0.1R_\odot} &= \frac{GM_\odot^2}{R_\odot}\left(\frac{1}{0.1} - 1\right) \\
&= \frac{9GM_\odot^2}{R_\odot} \\
&= \frac{9 \times (6.67 \times 10^{-11}\,\text{N m}^2\,\text{kg}^{-2}) \times (2 \times 10^{30}\,\text{kg})^2}{7 \times 10^{8}\,\text{m}} \\
&= 3 \times 10^{42}\,\text{J}
\end{aligned}$$

Assuming that this energy is all converted into power for the Sun, the time for which the Sun's present luminosity could be maintained is given by

$$\frac{3 \times 10^{42}\,\text{J}}{4 \times 10^{26}\,\text{W}} = 8 \times 10^{15}\,\text{s} = 3 \times 10^{8}\ \text{years}$$

Although much more promising than chemical energy (see Question 6.3) as an energy source, gravitational energy still can't provide sufficient energy to power the Sun for its known lifetime.

QUESTION 6.5

The electrical potential energy is found using Equation 6.8,

$$\begin{aligned}
E_\text{e} &= 8.99 \times 10^{9}\,\text{N m}^2\,\text{C}^{-2}\,(1 \times 1.6 \times 10^{-19}\,\text{C})(1 \times 1.6 \times 10^{-19}\,\text{C})/(10^{-15}\,\text{m}) \\
&= 2.3 \times 10^{-13}\,\text{J}
\end{aligned}$$

The kinetic energy due to thermal motion is found using Equation 4.2

$$\begin{aligned}
E_\text{k} &= 3\,(1.38 \times 10^{-23}\,\text{J K}^{-1})(1.6 \times 10^{7}\,\text{K})/2 \\
&= 3.3 \times 10^{-16}\,\text{J}
\end{aligned}$$

The average kinetic energy is therefore ~1000 times smaller than the electrical potential energy. So it can be concluded that the thermal motion of the particles is insufficient to overcome the electrostatic forces that act between protons.

QUESTION 6.6

The ratio of radiation pressure to gas pressure can be calculated from Equations 6.11 and 6.12 as

$$\frac{P_{\text{rad}}}{P_{\text{gas}}} = \frac{\alpha T^3}{3nk}$$

For the Sun, the average mass density is

$$\rho = \frac{1.99 \times 10^{30}\ \text{kg}}{\frac{4}{3}\pi(6.96 \times 10^8\ \text{m})^3} = 1.41 \times 10^3\ \text{kg m}^{-3}$$

The mean number density of hydrogen nuclei is

$$\frac{\text{mass density}}{\text{mass per particle}} = \frac{1.41 \times 10^3\ \text{kg m}^{-3}}{1.67 \times 10^{-27}\ \text{kg}}$$
$$= 8.44 \times 10^{29}\ \text{m}^{-3}$$

In the ionized solar interior, there are as many free electrons as there are hydrogen nuclei, so n (the number of particles per m^3) is double this value. Thus, using $T = 1.5 \times 10^7$ K for the centre of the Sun,

$$\frac{P_{\text{rad}}}{P_{\text{gas}}} = \frac{(7.55 \times 10^{-16}\ \text{kg m}^{-1}\ \text{s}^{-2}\ \text{K}^{-4}) \times (1.6 \times 10^7\ \text{K})^3}{3 \times (2 \times 8.44 \times 10^{29}\ \text{m}^{-3}) \times (1.38 \times 10^{-23}\ \text{kg m}^2\ \text{s}^{-2}\ \text{K}^{-1})}$$
$$= 4.4 \times 10^{-2}$$

So, according to this calculation, at the centre of the Sun radiation pressure is not very significant, amounting to about 4% of that due to gas pressure.

In Figure 2.1, we see that the mass density at the centre of the Sun is predicted to be about $1.5 \times 10^5\ \text{kg m}^{-3}$ compared with our average value of $1.4 \times 10^3\ \text{kg m}^{-3}$. So, we have underestimated the number density by about a factor of 100. This means that the contribution of radiation pressure is even less than calculated here (by a factor of 100).

QUESTION 6.7

Centre to surface of the Sun: radiation in the deep interior, then convection in the outer regions. (Conduction doesn't play a significant part in energy transport in most stars.)

Surface of the Sun to the top of the Earth's atmosphere: radiation, because this region is occupied by a vacuum.

Top of the atmosphere to the surface of the Earth: radiation (the atmosphere is partially transparent to some of the Sun's radiation). Although convection is an important process in energy transport in the atmosphere, it does not play a part in the initial transfer of energy to the surface as heat cannot be convected downwards.

QUESTION 6.8

We see from Table 6.1 that upper main sequence stars have main sequence lifetimes much less than the present age of the Earth. In the table, a lifetime of 4.5×10^9 years falls between the values for stars of mass $1M_\odot$ and $1.5M_\odot$, at a mass of approximately $1.3M_\odot$. Any star more massive than this, and that is now on the main sequence, cannot have been on the main sequence when the Earth formed.

QUESTION 6.9

(a) The reaction obeys the laws of conservation of electric charge and of baryon number. It will obey the law of conservation of energy provided that the sum of the rest energies and kinetic energies of the carbon nuclei equals this sum for the magnesium nucleus plus the energy of the γ-ray. The reaction is thus possible.

(b) Figure 6.6 shows that the rest energy *per nucleon* for $A = 12$ exceeds that for $A = 24$. Denoting these values by E_{12} and E_{24}, respectively, for the reactants we have a rest energy of $2 \times 12 \times E_{12}$, and for the product $24 \times E_{24}$. Thus, with $E_{12} > E_{24}$, the rest energy of the reactants exceeds that of the product, and so the reaction is exothermic. However, it is not an appreciable source of energy compared with the pp and CNO cycles, for main sequence stars because (i) with $Z = 6$ the electrical repulsion between the carbon nuclei results in a much lower reaction rate, and (ii) carbon is not nearly as abundant as hydrogen (or as helium). (This reaction is important however, in the post main sequence evolution of stars more massive than about $8–10M_\odot$.)

QUESTION 7.1

The luminosity of a star is related to its radius, R, and surface temperature, T, by Equation 3.9, i.e.

$$L \approx 4\pi R^2 \sigma T^4$$

where σ is the Stefan–Boltzmann constant. For the star we are considering here, we know that the star has expanded (i.e. R has increased) whereas the luminosity hasn't changed significantly (L is approximately constant). Thus, referring to Equation 3.9, it is required that T *decreases*, i.e. the surface of the star cools.

QUESTION 7.2

To answer this question it is necessary to determine whether the spectral types and luminosities of the stars in Table 7.1 correspond to positions on the H–R diagram that lie within the instability strip. The H–R diagram that shows the instability strip (Figure 7.6) is given in terms of temperature and luminosity. Thus the given spectral types and absolute magnitudes have to be first converted into temperatures and luminosities. This can be done by referring back to Table 3.2 and Figure 4.5. Table 7.2 shows the temperatures and luminosities that correspond to the spectral types and luminosities in Table 7.1. (Remember that the correspondence between absolute visual magnitude and luminosity is not exact.)

By plotting the values of luminosity and temperature on Figure 7.6, it can be seen that stars X and Y lie within the instability strip, whereas Z lies well outside it. Thus X and Y may be pulsating variables, whereas Z is unlikely to show large amplitude stellar pulsations.

Table 7.2 The spectral types and absolute magnitudes from Table 7.1 and the corresponding values of temperature and luminosity.

Star	Spectral type	Temperature/K	M_V	Luminosity/$L_\odot$
X	F0	7400	3	10
Y	K5	4100	–6	10^4
Z	A0	9900	1	10^2

QUESTION 7.3

In going to the right in the H–R diagram the surface temperature drops (considerably). The luminosity will rise a little if the star's evolution lifts it a little higher in the H–R diagram. (See, for example, the evolutionary track of a $15M_\odot$ star in Figure 7.2.)

QUESTION 7.4

Temperature and radius are related by Equation 3.9, $L \approx 4\pi R^2 \sigma T^4$.
Since the luminosity L is constant, and so are 4, π and σ, we can rearrange Equation 3.9 to give

$$R = \frac{1}{T^2}\left(\frac{L}{4\pi\sigma}\right)^{1/2} = \frac{\text{constant}}{T^2}$$

Therefore $$\frac{R_{\text{final}}}{R_{\text{initial}}} = \frac{(T_{\text{initial}})^2}{(T_{\text{final}})^2} = \left(\frac{25\,000}{5000}\right)^2$$

$$= 5^2 = 25$$

Therefore $$R_{\text{final}} = 25R_{\text{initial}}$$

$$= 25 \times 10R_\odot = 250R_\odot$$

In units of AU,

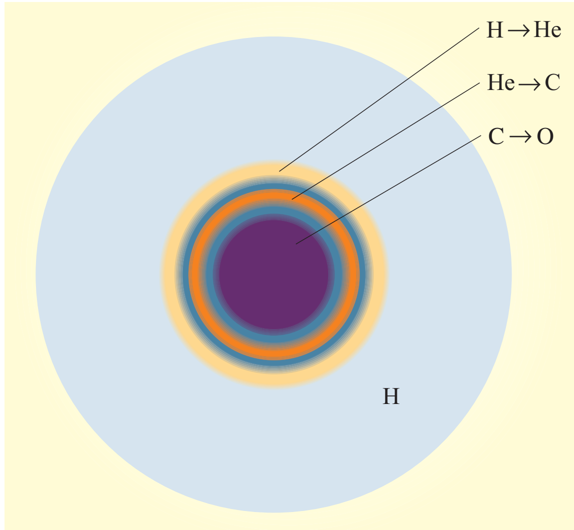
$$R_{\text{final}} = \frac{250R_\odot \times 7 \times 10^8 \text{ m } R_\odot^{-1}}{1.5 \times 10^{11} \text{ m AU}^{-1}}$$

$$= 1.2 \text{ AU}$$

QUESTION 7.5

One reason for there being so few stars in this area is that massive stars evolve very quickly from being upper main sequence stars to being supergiant stars, so the chances of catching a star in between are slender.

QUESTION 7.6



The answer is shown in Figure 7.14.

Figure 7.14 The structure of a star at the carbon-burning stage (not to scale).

QUESTION 7.7

In such a high-temperature environment, it is likely that any protons or α-particles would very rapidly undergo fusion reactions with the heavier nuclei present in the core. A reaction that may (and does) occur when α-particles are mixed with carbon at such extreme temperatures is:

$$^{12}_{6}\text{C} + {}^{4}_{2}\text{He} \rightarrow {}^{16}_{8}\text{O} + \gamma$$

Since these light nuclei would undergo reactions very quickly, no significant quantities of hydrogen or helium would build up in the core.

QUESTION 7.8

The wavelength of the peak of the black-body spectrum is given by Wien's displacement law (Equation 1.4):

$$\lambda_{\text{peak}} = \frac{2.90 \times 10^{-3}}{(T/\text{K})}\ \text{m} = \frac{2.90 \times 10^{-3}}{(10^{9})}\ \text{m} = 2.90 \times 10^{-12}\ \text{m}$$

So the wavelength of the peak of the black-body spectrum is at about 3×10^{-12} m. From Figure 1.36 it can be seen that this corresponds to the γ-ray part of the electromagnetic spectrum.

QUESTION 7.9

Initially, the fusion of hydrogen to form helium takes place in the core of the star. When the amount of available hydrogen there is appreciably reduced the reaction stops in the core. With the rise in temperature it can continue where there is hydrogen available at the edge of the core. So we find next that hydrogen is being fused to helium in a thin shell around the core. Eventually there is a shortage of hydrogen here too, and when the temperature rises again the site of the reaction moves outwards again to a fresh supply. Then the reaction is found in the next

layer out, i.e. in a shell of slightly larger radius surrounding both the previous shell and the core. This process is repeated over and over again, with the site of the hydrogen fusion reaction steadily moving outwards to shells of larger radius.

QUESTION 7.10

The mass lost from a solar mass main sequence star in 10^{10} years is simply

$$10^{10} \text{ years} \times 10^{-14} M_\odot \text{ year}^{-1} = 10^{-4} M_\odot$$

The fractional mass loss is therefore $10^{-4}/1$ or 0.01%.

For the $50M_\odot$ main sequence star the mass lost is

$$10^{6} \text{ years} \times 10^{-6} M_\odot \text{ year}^{-1} = 1 M_\odot$$

The fractional mass loss is therefore 1/50 or 2%.

Despite its short main sequence lifetime, the $50M_\odot$ star loses a larger fraction of its mass.

QUESTION 7.11

Your diagram should resemble Figure 7.15.

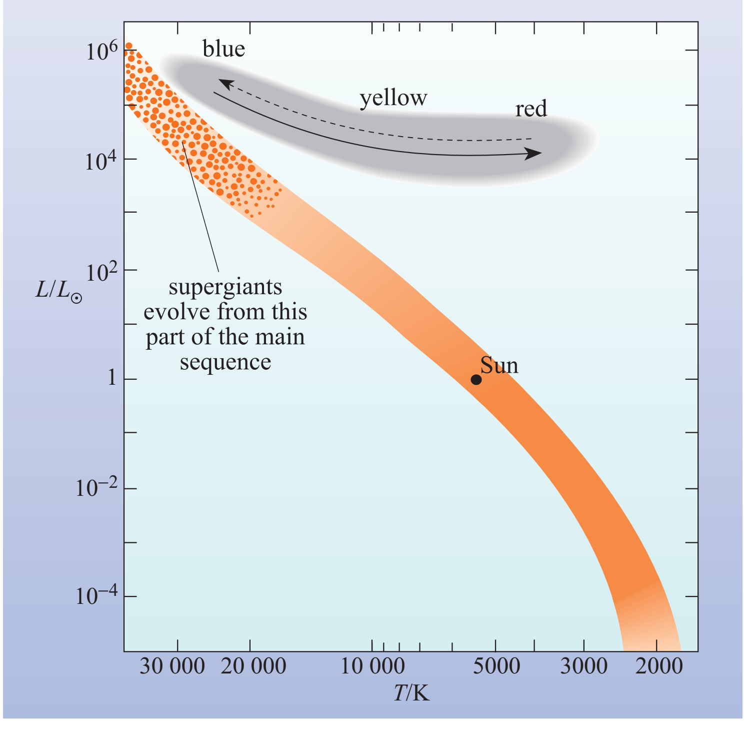


Figure 7.15 H–R diagram showing the positions of supergiants and an indication of their evolutionary tracks from blue to red (solid line) and back again (dashed line).

QUESTION 7.12

(a) For a $25M_\odot$ star, the correct chronological order of core nuclear burning processes is: hydrogen burning, helium burning, carbon burning, neon burning, oxygen burning and silicon burning.

(b) The processes that involve the fusion of two identical nuclei are: hydrogen, carbon and oxygen burning, whereas helium burning involves the fusion of three identical nuclei. Photodisintegration plays an important role in neon burning and silicon burning, as it provides a source of α-particles that can fuse with other nuclei.

QUESTION 7.13

There are three reasons why a massive star goes through its life cycle at an ever-increasing pace:

1 As the star evolves its central temperature rises; the nuclear reaction rates increase markedly as this happens.

2 As the reaction rates increase there is an increase in the emission of neutrinos; the neutrinos carry away energy that is then lost to the star and has to be replaced by further nuclear reactions; the star must contract and heat up so that the reaction rate increases to compensate for this increasing energy loss.

3 Heavier nuclei are produced as the star ages; the fusion of heavier nuclei produces less energy per kilogram than the fusion of lighter nuclei and so more of the heavy nuclei undergo fusion per second to maintain the necessary energy supply.

QUESTION 8.1

The density is equal to the mass divided by the volume, and so

$$3\times10^{17}\ \mathrm{kg\,m^{-3}} = \frac{6\times10^{24}\ \mathrm{kg}}{\text{volume}}$$

Therefore $\quad \text{volume} = \dfrac{6\times10^{24}}{3\times10^{17}}\ \mathrm{m^3} = 2\times10^{7}\ \mathrm{m^3}$

But volume $= \frac{4}{3}\pi r^3$, therefore

$$r = \left(\frac{3}{4\pi}\times2\times10^{7}\right)^{1/3}\ \mathrm{m} = (4.8\times10^{6})^{1/3}\ \mathrm{m} = 168\ \mathrm{m}\ (!)$$

So the radius would be 2×10^{2} m (to one significant figure).

QUESTION 8.2

A supergiant star might have a radius of 100 times the radius of the Sun (Section 4.2.1), that is $100\times7\times10^{5}$ km, which is 7×10^{7} km. Our expanding supernova has a radius of 2×10^{10} km which is about $2\times10^{10}\ \mathrm{km}/7\times10^{7}\ \mathrm{km} \approx 3\times10^{3}$ times bigger than such a supergiant. Pluto, the outermost planet in our Solar System, is approximately 6×10^{9} km from the Sun; taking this as the radius of the Solar System, we see that the supernova has expanded to about three times this size. The distance to the nearest star is 1.30 parsecs, which is roughly 4×10^{13} km; the supernova has (so far) expanded to only one two-thousandth of this.

QUESTION 8.3

Neutrinos travel readily through the Earth, so it is not necessary for the source to be above the horizon. In this case one would suspect that some, if not all, of the neutrinos had travelled through some of the Earth before detection because (i) Japan and Ohio are approximately on opposite sides of the world, so it is unlikely that a source that they detect simultaneously would be above both their horizons at that time, and (ii) the Large Magellanic Cloud is in the southern sky and these detectors are in the northern hemisphere, so it might well always be below both their horizons. (In fact, this is case – the Large Magellanic Cloud never rises above the horizon for observers in Ohio or Japan.)

QUESTION 8.4

If there are two γ-rays of energy 1.3×10^{-13} J and 1.9×10^{-13} J, then the energy output per decay = 3.2×10^{-13} J. To provide a luminosity of 10^{33} W, i.e. $10^{33}\,\mathrm{J\,s^{-1}}$,

$$\text{the rate of decay} = \frac{10^{33}\ \mathrm{J\,s^{-1}}}{3.2\times10^{-13}\ \mathrm{J}} = 3\times10^{45}\ \text{decays s}^{-1}$$

Each ${}^{56}_{27}\mathrm{Co}$ nucleus weighs about 56 times the mass of a neutron or proton, which is $56 \times 1.7 \times 10^{-27}$ kg. Therefore

$$\begin{aligned}\text{mass decaying per second} &= (56\times1.7\times10^{-27}\ \mathrm{kg})\times(3\times10^{45}\ \mathrm{s^{-1}})\\ &= 3\times10^{20}\ \mathrm{kg\,s^{-1}}\\ &= 1.5\times10^{-10}\,M_\odot\ \mathrm{s^{-1}}\end{aligned}$$

The cobalt γ-rays illuminate the remnant for hundreds of days, so clearly a large mass of nickel (which decayed into the cobalt) was created. It is estimated that, in SN 1987A, $0.07M_\odot$ of nickel was formed.

QUESTION 8.5

The kinetic energy of the shell is given by

$$E_\mathrm{k} = \tfrac{1}{2}Mv^2$$

where M is the mass of the shell and v is the required speed. Thus

$$v = (2E_\mathrm{k}/M)^{1/2}$$

With $E_\mathrm{k} \sim 10^{44}$ J, and $M \sim 0.25M_\odot \sim 5 \times 10^{29}$ kg, we get

$$v \approx \left(\frac{2\times10^{44}\ \mathrm{J}}{5\times10^{29}\ \mathrm{kg}}\right)^{1/2}$$

Replacing J by the equivalent units $\mathrm{kg\,m^2\,s^{-2}}$, we get

$$\begin{aligned}v &\approx \left(\frac{2\times10^{44}\ \mathrm{kg\,m^2\,s^{-2}}}{5\times10^{29}\ \mathrm{kg}}\right)^{1/2}\\ &\approx (4\times10^{14}\ \mathrm{m^2\,s^{-2}})^{1/2}\\ &\approx 2\times10^{7}\ \mathrm{m\,s^{-1}} \approx 0.07c\end{aligned}$$

QUESTION 8.6

(a) For a thermal source of radiation, a rough estimate of the photon energy at which the emission will be relatively strong can be made using the relation (Equation 1.6):

$$\varepsilon \sim kT$$

$$\varepsilon \sim 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}} \times 10^{6}\,\mathrm{K} = 1.38 \times 10^{-17}\,\mathrm{J}$$

Converting into electronvolts,

$$\varepsilon \sim (1.38 \times 10^{-17}\,\mathrm{J})/(1.60 \times 10^{-19}\,\mathrm{J\,eV^{-1}}) = 86.3\,\mathrm{eV}$$

Thus the expected photon energy is about 90 eV. (Remember that this equation gives only a rough estimate of the expected photon energy, and that the answer should only be quoted to 1 significant figure.)

(b) The wavelength corresponding to a photon of this energy is found by combining Equations 1.2 and 1.3 (see Example 1.1)

$$\lambda = hc/\varepsilon$$

$$\lambda = (6.63 \times 10^{-34}\,\mathrm{J\,s})\,(3.00 \times 10^{8}\,\mathrm{m\,s^{-1}})/(1.38 \times 10^{-17}\,\mathrm{J})$$

$$= 1.44 \times 10^{-8}\,\mathrm{m}$$

So the wavelength at which strong thermal emission would be expected is 1×10^{-8} m. (Again, the answer is a rough estimate and should only be quoted to 1 significant figure.)

From Figure 1.36, it can be seen that such a wavelength corresponds to the X-ray regime of the electromagnetic spectrum.

QUESTION 8.7

If it is assumed that the outflow speed has remained constant since the planetary nebula formed, then the age can be calculated from

$$\text{age} = \frac{\text{radius}}{\text{expansion speed}} = \frac{(0.4/2) \times 3.09 \times 10^{16}\ \mathrm{m}}{20 \times 10^{3}\ \mathrm{m\,s^{-1}}}$$

$$= 3.09 \times 10^{11}\ \mathrm{s}$$

Converting this into years

$$\text{age} = (3.09 \times 10^{11})/(365 \times 24 \times 60 \times 60)\ \text{years} = 9.80 \times 10^{3}\ \text{years}$$

So to 1 significant figure, the age of the planetary nebula is 1×10^{4} years.

QUESTION 8.8

The flux density F depends on luminosity L and distance d as given by Equation 3.10:

$$F = \frac{L}{4\pi d^{2}}$$

If the flux density from the supernova equals that from the Sun, then

$$\frac{L_{\odot}}{4\pi d_{\odot}^{2}} = \frac{L_{\mathrm{SN}}}{4\pi d_{\mathrm{SN}}^{2}}$$

where SN denotes the supernova. Thus

$$d_{SN}^2 = \left(\frac{L_{SN}}{L_\odot}\right) d_\odot^2$$

Now, we are given that $(L_{SN}/L_\odot) = 5 \times 10^9$, and we know that

$$d_\odot = 1.0 \text{ AU} = 4.9 \times 10^{-6} \text{ pc}$$

Therefore $d_{SN} = (5 \times 10^9)^{1/2} \times 1.0$ AU

$\approx 7 \times 10^4$ AU

or $d_{SN} = (5 \times 10^9)^{1/2} \times 4.9 \times 10^{-6}$ pc

≈ 0.3 pc

There are no supergiants that close! The nearest known star (of any sort) after the Sun is 1.3 parsecs away (Section 3.2.2).

QUESTION 8.9

The answer is a diagram – Figure 8.20.

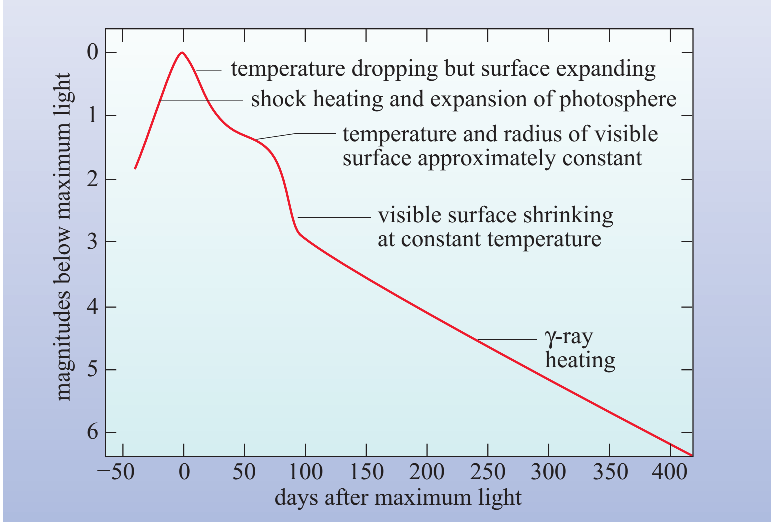


Figure 8.20 For the answer to Question 8.9.

QUESTION 8.10

It can be seen from Table 8.2, that addition of neutrons by the s-process to the stable isotopes of palladium with mass numbers 104, 105 and 106 will successively form isotopes with mass numbers 105, 106 and 107. Table 8.2 shows that $^{107}_{46}\text{Pd}$ is unstable, and so it is necessary to consider whether the nuclei of this isotope will decay before the next neutron capture event. The half-life for the β^--decay is 6.5×10^6 years. This is far longer than the typical interval between s-process neutron capture events, which is about 10^4 years. So the nuclei of $^{107}_{46}\text{Pd}$ are much more likely to capture another neutron than to undergo β^--decay. This means that the isotope of palladium with mass number 108 *is* likely to be formed by the s-process. This is a stable isotope and so can also capture a neutron by the

s-process to form ${}^{109}_{46}\mathrm{Pd}$. This is an unstable isotope with a half-life of only 13.7 hours, and so nuclei of ${}^{109}_{46}\mathrm{Pd}$ will undergo β^--decay (to form an isotope of silver) before neutron capture can occur. Thus, it is impossible to form the isotope ${}^{110}_{46}\mathrm{Pd}$ by the s-process. Thus the isotopes of palladium with mass numbers 105 to 109 can be formed by the s-process, whereas the isotope with mass number 110 cannot.

QUESTION 8.11

Stellar winds and the diffusion of old planetary nebulae would be the main ways in which material was returned to the interstellar medium. Both these, however, are from the outermost layers of the star, and are predominantly hydrogen. Small amounts of dust (silicates and graphite) would continue to diffuse into the interstellar medium from the dust shells found surrounding some stars. A few stars have deep convection currents that pull material up to the surface, but in most cases the elements created in stars would remain locked in the stars. Thus there would be a lower abundance of elements heavier than helium. Elements that can be created only by the r-process would not exist.

QUESTION 9.1

The density ρ is calculated from ρ = mass/volume. For a spherical body, with mass M and radius R,

$$\rho = \frac{M}{\frac{4}{3}\pi R^3}$$

For a white dwarf with

$$M = 0.6M_\odot = 0.6 \times 2.0 \times 10^{30}\,\mathrm{kg} = 1.2 \times 10^{30}\,\mathrm{kg}$$

and $R = 7000\,\mathrm{km} = 7 \times 10^6\,\mathrm{m}$

the density ρ_{WD} is given by

$$\rho_{\mathrm{WD}} = \frac{2.0 \times 10^{30}\ \mathrm{kg}}{\frac{4}{3}\pi(7 \times 10^6\ \mathrm{m})^3} = 8.4 \times 10^8\ \mathrm{kg\,m^{-3}}$$

To one significant figure the density of the white dwarf is $8 \times 10^8\,\mathrm{kg\,m^{-3}}$.

For the Sun, the average density $\rho_\odot$ is given by

$$\rho_\odot = \frac{2.0 \times 10^{30}\ \mathrm{kg}}{\frac{4}{3}\pi(7 \times 10^8\ \mathrm{m})^3} = 1.4 \times 10^3\ \mathrm{kg\,m^{-3}}$$

Thus, the white dwarf has a density that is about 6×10^5 times greater than that of the Sun.

QUESTION 9.2

From Equation 3.9, we know that the luminosity of a star is given by

$$L = 4\pi R^2 \sigma T^4$$

We can write, for the case of two different luminosities at constant radius

$$L_1 = 4\pi R^2 \sigma T_1^4 \quad \text{(i)}$$

$$L_2 = 4\pi R^2 \sigma T_2^4 \quad \text{(ii)}$$

Therefore, by dividing Equation (i) by Equation (ii),

$$\left(\frac{L_1}{L_2}\right) = \left(\frac{T_1}{T_2}\right)^4$$

In this case, $L_2 = 10^{-4} L_1$, (i.e. $L_1/L_2 = 10^4$)

$$10^4 = \left(\frac{T_1}{T_2}\right)^4$$

and so $T_1/T_2 = 10$

In other words, the surface temperature has dropped by a factor of 10. We can check this by referring to Figure 9.1. If we follow the evolutionary track shown for a white dwarf, a drop in luminosity from, say, $1L_\odot$ to $10^{-4} L_\odot$ (in other words, a reduction by a factor of 10^4), corresponds to a change in surface temperature from about 60 000 K to about 6000 K. This confirms that a drop in luminosity by a factor of 10^4 corresponds to a drop in surface temperature by a factor of 10.

QUESTION 9.3

It is best to express the star's mass in kilograms:

$$1.5M_\odot = 3.0 \times 10^{30}\,\text{kg}$$

and the radius in metres:

radius (r) = 10 km = 10^4 m

From the radius, we can get the volume:

$$\text{volume} = \tfrac{4}{3}\pi r^3 \text{ (assuming the star is spherical)}$$

$$= \tfrac{4}{3}\pi(10^4)^3\,\text{m}^3 = 4.2 \times 10^{12}\,\text{m}^3$$

$$\text{density} = \text{mass/volume}$$

Therefore $$= \frac{3.0 \times 10^{30}}{4.2 \times 10^{12}}\,\text{kg}\,\text{m}^{-3} = 7.1 \times 10^{17}\,\text{kg}\,\text{m}^{-3}$$

We can take the volume of a thimble as 1 $\text{cm}^3 = 10^{-6}\,\text{m}^3$, so the mass of a thimble-full, M_t, is

$$M_\text{t} = 10^{-6} \times 7.1 \times 10^{17}\,\text{kg} = 7.1 \times 10^{11}\,\text{kg}$$
$$= 7.1 \times 10^8 \text{ tonnes}$$

So a thimble-full of neutron star material will have a mass of 7×10^8 tonnes.

QUESTION 9.4

(a) The acceleration due to gravity at the surface of the Earth is given by

$$g_E = \frac{GM_E}{R_E^2}$$

where M_E is the mass of the Earth and R_E its radius. Using the values for mass and radius given in the question, (mass = 5.98×10^{24} kg, radius = 6378 km),

$$g_E = \frac{(6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}) \times (5.98 \times 10^{24}\ \mathrm{kg})}{(6.378 \times 10^6\ \mathrm{m})^2}$$

$$= 9.805\ \mathrm{m\,s^{-2}}$$

So the acceleration due to gravity at the surface of the Earth is $9.81\ \mathrm{m\,s^{-2}}$ (to 3 significant figures).

(b) The acceleration due to gravity at the surface of a neutron star is

$$g_n = \frac{GM_n}{R_n^2}$$

so, with $M_n = 1.5M_\odot = 3.0 \times 10^{30}$ kg and $R_n = 10\ \mathrm{km} = 10^4$ m,

$$g_n = \frac{(6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}) \times (3.0 \times 10^{30}\ \mathrm{kg})}{(10^4\ \mathrm{m})^2}$$

$$= 2.00 \times 10^{12}\ \mathrm{m\,s^{-2}}$$

The acceleration due to gravity at the surface of this neutron star is $2.0 \times 10^{12}\ \mathrm{m\,s^{-2}}$ (to 2 significant figures).

QUESTION 9.5

The speed v of an object that is initially at rest, if it has undergone constant acceleration a for a time t is

$$v = at$$

In this case, $t = 1.0 \times 10^{-5}$ s, and the accelerations are those found in Question 9.4

(a) For an object that is dropped near the surface of the Earth,

$$v = 9.81 \times 1.0 \times 10^{-5}\ \mathrm{m\,s^{-1}}$$

The speed of the object after 1.0×10^{-5} s is $9.8 \times 10^{-5}\ \mathrm{m\,s^{-1}}$ (to 2 significant figures).

(b) For an object that is dropped near the surface of the neutron star,

$$v = 2.0 \times 10^{12} \times 1.0 \times 10^{-5}\ \mathrm{m\,s^{-1}}$$

Therefore the speed of the object after 1.0×10^{-5} s is $2.0 \times 10^7\ \mathrm{m\,s^{-1}}$ (to 2 significant figures).

This is about 7% of the speed of light, and is an indication that a full description of gravitational effects near a neutron star require the use of Einstein's General Theory of Relativity rather than Newton's theory of gravity.

QUESTION 9.6

(a) A full rotation corresponds to an angle of 2π radians. Hence a body that rotates through a half turn in one second, turns through an angle of π radians per second. The angular speed is then π rad s^{-1} (or equivalently 3.14 rad s^{-1}).

(b) The period of rotation is 0.25 s, so using Equation 9.1, the angular speed is

$$\omega = 2\pi/T = 2\pi/0.25 = 8\pi \text{ rad s}^{-1} \text{ (or equivalently } \omega = 25.1 \text{ rad s}^{-1}\text{)}$$

(c) The period of rotation is 4.0 s, so again using Equation 9.1, the angular speed is

$$\omega = 2\pi/T = 2\pi/4.0 \text{ rad s}^{-1} = \pi/2 \text{ rad s}^{-1} \text{ (or equivalently } \omega = 1.57 \text{ rad s}^{-1}\text{)}$$

(d) The frequency is 4.0 Hz. Using Equation 9.2 the angular speed is

$$\omega = 2\pi f = 2\pi \times 4.0 \text{ rad s}^{-1} = 8\pi \text{ rad s}^{-1} \text{ (or equivalently } \omega = 25.1 \text{ rad s}^{-1}\text{).}$$

Note that in all these cases, the answer may be expressed in terms of π. This is often a convenient way of expressing an angular speed. For instance, in converting from an angular speed to a frequency, we have to divide the angular speed by 2π; this calculation is very straightforward if angular speed is expressed as a multiple of π.

QUESTION 9.7

This calculation is analogous to calculations about the brightness of stars at various distances, and similarly we have to remember that the flux density received diminishes as $1/d^2$, where d is the distance to the source. We need to compare the ratio of radio luminosity to d^2 for the transmitter and the pulsar.

$$\text{flux density from transmitter} = \frac{L_t}{4\pi d_t^2}$$

$$\text{flux density from pulsar} = \frac{L_p}{4\pi d_p^2}$$

Therefore

$$\begin{aligned}\frac{\text{flux density from transmitter}}{\text{flux density from pulsar}} &= \frac{L_t}{L_p}\left(\frac{d_p}{d_t}\right)^2 \\ &= \frac{10^5 \text{ W}}{10^{20} \text{ W}}\left(\frac{10^4 \times 3 \times 10^{16} \text{ m}}{10^5 \text{ m}}\right)^2 \\ &= 10^{-15} \times 9 \times 10^{30} = 9 \times 10^{15} \\ &\approx 10^{16}\end{aligned}$$

Therefore the flux density from the radio transmitter is 10^{16} times stronger than that from the pulsar.

This highlights one of the problems of radio astronomy – the cosmic signals are weak and can easily be swamped by terrestrial radio signals if they stray onto the channels reserved for radio astronomy.

QUESTION 9.8

An accuracy of 1 part in 10^{14} means, for example, that the accuracy can be expressed as 1 second in 10^{14} seconds.

$$1 \text{ century} = 100 \text{ years} = 100 \times 3.2 \times 10^7 \text{ seconds} = 3.2 \times 10^9 \text{ s}$$

Therefore the number of seconds accuracy per century

$$= 3.2 \times 10^9/10^{14} \text{ s} = 3.2 \times 10^{-5} \text{ s} = 32 \text{ microseconds}$$

QUESTION 9.9

(a) The wavelength of the peak of emission of the black-body spectrum is given by Wien's displacement law (Equation 1.4). In this case $T = 5 \times 10^5$ K, so

$$(\lambda_{\text{peak}}/\text{m}) = \left(\frac{2.90 \times 10^{-3}}{5 \times 10^5}\right)$$

$$\lambda_{\text{peak}} = 5.80 \times 10^{-9} \text{ m}$$

The peak of the emission occurs at about 6×10^{-9} m, and according to Figure 1.36, this corresponds to a band between X-ray and ultraviolet regimes of the electromagnetic spectrum. This regime is sometimes referred to as a soft X-ray band or extreme-UV.

(b) The luminosity of the neutron star is given by Equation 3.9

$$L = 4\pi R^2 \sigma T^4$$
$$= 4\pi\,(1 \times 10^4 \text{ m})^2 \times (5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}) \times (5 \times 10^5 \text{ K})^4$$
$$= 4.45 \times 10^{24} \text{ W}$$

In terms of the solar luminosity, this is

$$L/L_\odot = (4.45 \times 10^{24} \text{ W})/(3.84 \times 10^{26} \text{ W}) = 1.2 \times 10^{-2}$$

So the luminosity of the neutron star will be about 1% of the solar luminosity.

QUESTION 9.10

The magnitude of the escape speed is given by

$$v_{\text{esc}} = \sqrt{2GM/R}$$

The escape speed is equal to c at $R = R_\text{S}$, so

$$c = \sqrt{2GM/R_\text{S}}$$

$$c^2 = 2GM/R_\text{S}$$

Therefore

$$R_\text{S} = \frac{2GM}{c^2}$$

This is an important equation, which gives the Schwarzschild radius of a black hole of mass M. We have derived it without using relativity, so the derivation is somewhat artificial. However, it does give the right result, and it does illustrate some of the physical processes that we have to consider.

QUESTION 9.11

Using the relationship derived in Question 9.10,

$$R_S = \frac{2\times(6.7\times10^{-11}\ \mathrm{N\,m^2\,kg^{-2}})\times(2.0\times10^{30}\ \mathrm{kg})}{(3.0\times10^{8}\ \mathrm{m\,s^{-1}})^2}$$
$$= 3.0\times10^{3}\ \mathrm{N\,kg^{-1}\,s^{2}}$$
$$= 3.0\times10^{3}\ \mathrm{kg\,m\,s^{-2}\,kg^{-1}s^{2}} = 3.0\times10^{3}\ \mathrm{m}$$
$$= 3.0\,\mathrm{km}$$

QUESTION 9.12

(a) The answer is a diagram. See Figure 9.25.

In comparing the magnitudes of the two gravitational forces we shall work with the masses of the objects in $M_\odot$ and distances in $R_\odot$:

$$F_g = \frac{GMm}{R^2}$$

Gravitational force due to the main sequence star:

$$F_{MS} = \frac{G(0.70M_\odot)\,m_{blob}}{(0.96R_\odot)^2}$$

Gravitational force due to the white dwarf:

$$F_{WD} = \frac{G(1.20M_\odot)\,m_{blob}}{(2.18R_\odot - 0.96R_\odot)^2}$$

$$\frac{F_{WD}}{F_{MS}} = \frac{G(1.20M_\odot)\,m_{blob}}{(2.18R_\odot - 0.96R_\odot)^2}\times\frac{(0.96R_\odot)^2}{G(0.70M_\odot)\,m_{blob}}$$
$$= \left(\frac{1.20}{0.70}\right)\times\left(\frac{0.96}{2.18-0.96}\right)^2 = 1.06$$

and so F_{WD} is slightly larger than F_{MS}.

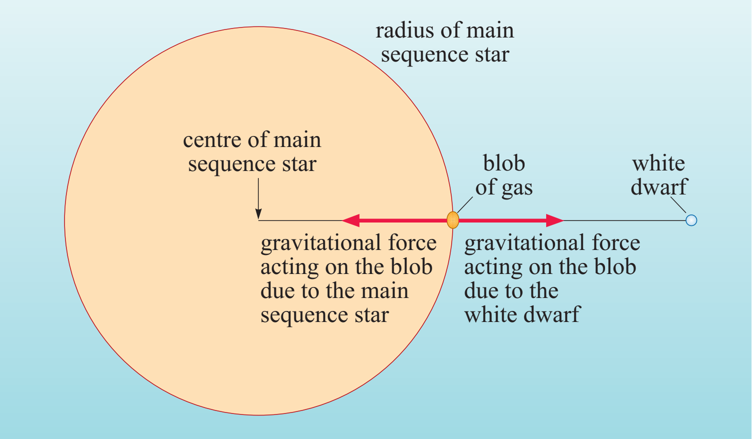


Figure 9.25 A diagram of the binary system described in Question 9.12, showing the direction of forces acting on the blob of gas under consideration.

QUESTION 9.13

Most of the energy released in a Type II supernova is in the form of neutrinos that accompany the collapse of the core of the star. This is about a factor of 100 greater than the energy released as kinetic energy of the ejecta and in electromagnetic radiation. Type Ia supernovae do not undergo core collapse, and there is no corresponding loss of energy by neutrinos. Hence the total energy released in a Type Ia supernova is only about 1% of the energy released in a Type II supernova.

QUESTION 9.14

If angular momentum is conserved, then $L_1 = L_2$, where the subscript 1 refers to the satellite before the astronaut arrives, and subscript 2 to the satellite–astronaut combination. Considering first the satellite alone:

$$L_1 = I_1\omega_1$$

and $I_1 = 2500\,\mathrm{kg\,m^2}$. The satellite spins at 0.33 revolutions per second, therefore the angular speed is (Equation 9.2)

$$\omega_1 = 2\pi \times 0.33\,\mathrm{rad\,s^{-1}} = 0.66 \times \pi\,\mathrm{rad\,s^{-1}}$$

(The factor π is left as a symbol, since later on, we will be required to divide by a factor 2π to get an answer in terms of the frequency.)

$$L_1 = (2500\,\mathrm{kg\,m^2}) \times (0.66 \times \pi\,\mathrm{rad\,s^{-1}})$$

$$= 1.65 \times 10^3 \times \pi\,\mathrm{kg\,m^2\,rad\,s^{-1}}$$

The astronaut has a moment of inertia about the spin axis of $100\,\mathrm{kg} \times (1\,\mathrm{m})^2 = 100\,\mathrm{kg\,m^2}$, so the satellite–astronaut combination has a moment of inertia of $(2500 + 100)\,\mathrm{kg\,m^2}$; therefore

$$I_2 = 2600\,\mathrm{kg\,m^2}$$

Now, since

$$L_2 = I_2\,\omega_2$$

$$\omega_2 = \frac{L_2}{I_2}$$

But $L_2 = L_1$ so

$$\omega_2 = \frac{L_1}{I_2}$$

$$= \frac{1.65 \times 10^3 \times \pi\,\mathrm{kg\,m^2\,rad\,s^{-1}}}{2600\,\mathrm{kg\,m^2}}$$

$$= 0.635 \times \pi\,\mathrm{rad\,s^{-1}}$$

This can be converted from an angular speed to an angular frequency by rearranging Equation 9.2

$$f = \frac{\omega_2}{2\pi} = \frac{0.635 \times \pi}{2\pi} = 0.318\,\mathrm{s^{-1}}$$

So the frequency of rotation drops to 0.32 revolutions per second.

QUESTION 9.15

The power received from the pulsar is

$$10^{-19}\ \text{W m}^{-2} \times 1000\ \text{m}^2 = 1 \times 10^{-16}\ \text{W}$$

The change in gravitational potential energy when lifting the book is

$$\begin{aligned}\Delta E_g &= mg\Delta h \\ &= 1\ \text{kg} \times 10\ \text{m s}^{-2} \times 1\ \text{m} \\ &= 10\ \text{J}\end{aligned}$$

$$\text{power used} = \Delta E_g/\text{time taken} = 10\ \text{J}/1\ \text{s} = 10\ \text{W}$$

Therefore $\dfrac{\text{power used in lifting book}}{\text{power received from pulsar}} = \dfrac{10\ \text{W}}{1 \times 10^{-16}\ \text{W}} = 10^{17}$

It can be concluded that pulsar signals are weak!

QUESTION 9.16

The B star and the pulsar will be moving around each other. The optical astronomer will therefore expect there to be changing Doppler shifts in the observed frequencies of the emission lines from the B star. The optical astronomer will not see the pulsar.

The radio astronomer will see similar changes in the pulsar period: owing to Doppler shifts the pulse period will be shorter than average for part of the orbit, and for part of the orbit it will be longer. The radio astronomer will not see the B star.

Assuming these two stars were formed as a binary pair, then the star that is now the pulsar (the neutron star) must originally have been the more massive and be more rapidly evolving. To become a neutron star it must have passed through its main sequence stage and its supergiant stage and then exploded as a supernova. Its core became the pulsar. Meanwhile the other star was more slowly going through the main sequence stages, and is now presumably forming heavier elements in its core. There cannot at the moment be much transfer of material from the B star to the pulsar (because if there were, this material would blanket the pulsar and prevent the radio emission).

We are not given any information about the size of the binary system. It may be that the stars are sufficiently far apart that they have not significantly affected each other. Or it could be that the B star received material from its companion when it was a supergiant. It could also be that the mass transfer from the B star to the neutron star has not yet begun because the B star has yet to swell sufficiently to fill its Roche lobe. If mass transfer does start, the radio emission will cease and X-ray emission begin. The rotation rate of the neutron star may be increased by the transferred material.

If there is a lot of mass transferred from the B star its evolution could be seriously affected, e.g. it might lose all its outer envelope. If the mass transfer is limited the B star will probably still pass through the supernova stage, perhaps with its core becoming another neutron star.